7. EXTENSIVE FORM GAMES

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- A **pure strategy** is like a "book" in which every "page" contains an instruction for one particular information set (and the book has precisely as many pages as the player has information sets). The instruction for a particular information set, of course, specifies one (but only one) of the choices available at this information set.

- Remember that a **Subgame Perfect Nash Equilibrium** is a combination of strategies that yield a Nash equilibrium in every subgame, whether these subgames are reached in equilibrium or not. If players' strategies constitute a Nash equilibrium in every subgame they specify moves that are best responses to each other in every subgame.

- Remember that we can use the **Backward Induction** to find the subgame perfect Nash equilibria in an extensive form game.

Exercise 1.

Step 1 First note that there are two proper subgames; see Figure 0.1.

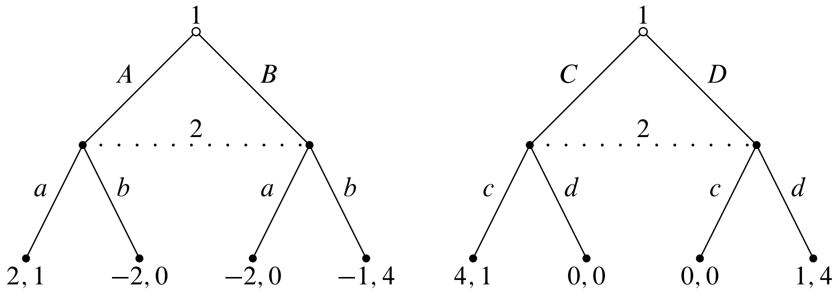

Figure 0.1: Two Subgames

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Step 2 We can use the following normal form games to represent the above two extensive form subgames; see Subgame 1 and Subgame 2.

		Player 2	
		a	b
Player 1	A	2 1	−2 0
	B	−2 0	−1 4

Subgame 1

		Player 2	
		c	d
Player 1	C	4 1	0 0
	D	0 0	1 4

Subgame 2

Step 3 In Subgame 1, the set of Nash equilibria is

$$NE^1 = \left\{ (A, a), (B, b), \left(\frac{4}{5}A + \frac{1}{5}B, \frac{1}{5}a + \frac{4}{5}b \right) \right\}.$$

In Subgame 2, the set of Nash equilibria is

$$NE^2 = \left\{ (C, c), (D, c), \left(\frac{4}{5}C + \frac{1}{5}D, \frac{1}{5}c + \frac{4}{5}d \right) \right\}.$$

Step 4 There are 9 cases need to be considered; for example, the NE in subgame 1 is (A, a), and the NE in subgame 2 is (C, c). In this case, the game becomes as in Figure 0.2, where Player 1 plays R since $4 > 2$, and the SPNE is

$$\left(\underline{R, A, C} \diamond \underline{a, c} \right).$$

I also depict the *path of play* from the above SPNE in Figure 0.3.

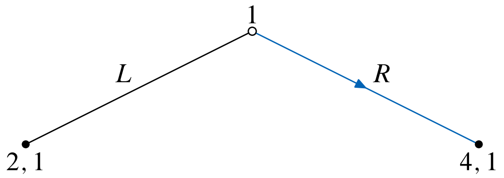

Figure 0.2: Case 1

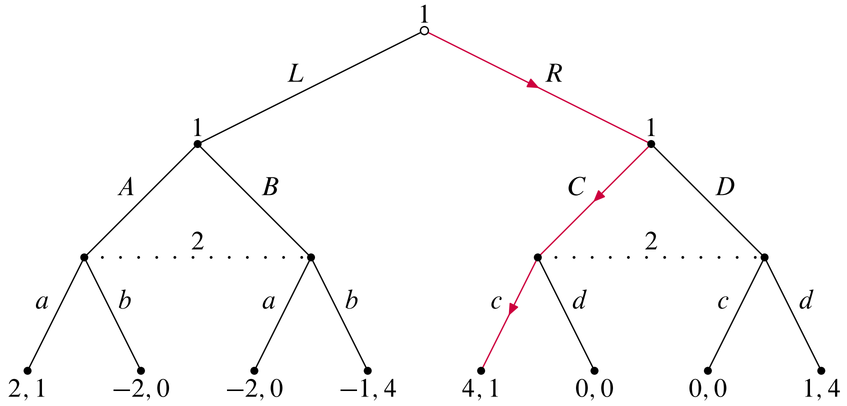

Figure 0.3: Path of Play in SPNE $\left(\underline{R, A, C} \diamond \underline{a, c}\right)$

Step 5 Now we consider another case, where the NE in subgame 1 is $\left(\frac{4}{5}A + \frac{1}{5}B, \frac{1}{5}a + \frac{4}{5}b\right)$, and the NE in subgame 2 is $\left(\frac{4}{5}C + \frac{1}{5}D, \frac{1}{5}c + \frac{4}{5}d\right)$. With these NE's, the corresponding pairs of payoffs are

$$\left(2 \times \frac{1}{5} + (-2) \times \frac{4}{5} = -\frac{6}{5}, 1 \times \frac{4}{5} + 0 \times \frac{1}{5} = \frac{4}{5}\right), \quad \text{and} \quad \left(\frac{4}{5}, \frac{4}{5}\right).$$

Thus, the whole game becomes as in Figure 0.4. In this case, Player 1 plays R since $\frac{4}{5} > -\frac{6}{5}$, and the SPNE is

$$\left(\underline{R, \frac{4}{5}A + \frac{1}{5}B, \frac{4}{5}C + \frac{1}{5}D,} \diamond \underline{\frac{1}{5}a + \frac{4}{5}b, \frac{1}{5}c + \frac{4}{5}d}\right).$$

You need to consider the other 7 cases.

Exercise 2.

Step 1 Please draw the unique proper subgame.

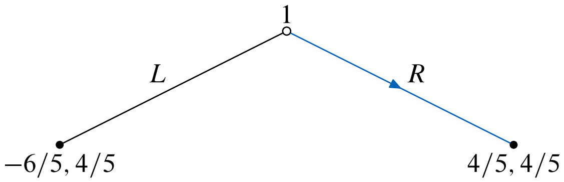

Figure 0.4: Case 2

Step 2 Please write down the strategic form game corresponding the above subgame.

Step 3 Find the set of Nash equilibria in the strategic form game.

Step 4 Write down the reduced games and find out the set of SPNE's [there are 3 cases].

Exercise 3.

Remark. For a player's strategy to be an equilibrium strategy in a repeated game it needs to be a best response to the other player's move in every repetition of the game. As each repetition is effectively a subgame of the whole game the appropriate equilibrium concept is that of a *subgame perfect Nash equilibrium*. In finitely repeated games there is a unique endgame and therefore we can use *backward induction* to find the subgame perfect Nash equilibrium of these kinds of games.

Remark. Consider the prisoners' dilemma in Matrix "Prisons' Dilemma". In the one-shot version of this game the dominant strategy equilibrium is (Confess, Confess). To solve the finitely repeated version of this prisoners' dilemma we can use backward induction to predict what the players will do in the last state, the last repetition of the game, and then work backwards through each stage of the game in the same way until we reach the first repetition of the game.

		Player 2	
		Not Confess	*Confess*
Player 1	*Not Confess*	10 10	0 11
	Confess	11 0	3 3

Prisons' Dilemma

The game is repeated 2 times.

Step 1 Using backward induction means that we start by analysing the moves of the players in the second repetition, the last round of the game. If the players reach the last stage they know that they are not going to play the game ever again and therefore the game looks like a one-shot game. Rational players will treat it as such and will defect as $(Confess, Confess)$ is the unique Nash equilibrium of the single-stage version of the game.

Step 2 In stage 1, rational players will be able to reason that they will confess in the last repetition of the game and therefore both players' dominant strategy will be to confess.

It does not matter the number of repetitions as long as the number is finite. We will talk about it in class.