CONTRIBUTION-BASED PUBLIC GOODS PROVISION WITH HETEROGENOUS ENDOWMENTS

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In an environment of voluntarily provision of public goods, we show that if the players are grouped according to their contributions and if their abilities to contribute are different, then there exist two positive contribution equilibria: one is fully efficient in the sense that all individuals contribute fully, and the other is near fully efficient in the sense that almost all individuals contribute fully.

KEYWORDS. Public goods; Group; Heterogeneous endowment; Free-rider; Selective incentives; Nash equilibrium.

JEL CLASSIFICATION. C72; H41.

1. INTRODUCTION

Groups or organizations are a means of carrying out most social, political, or economic activities. After all, a society is an organization of organizations (North, Wallis and Weingast, 2009). Broadly speaking, organizations include labor unions, professional associations, farm organizations, cartels, lobbies, universities, government, and so on. Presumably, groups or organizations increase the benefits of collective action in situations in which the price system fails (Arrow, 1974). Furthermore, social groups are treated as key political actors because the most important forces in political conflict and change are groups of individuals (Acemoglu and Robinson 2006 and Besley and Persson 2011). As Coleman (1974) writes:

> It is the corporate actors, the organizations that draw their power from persons and employ that power to corporate ends, that are the primary actors in the social structure of modern society (1974, p. 49).

What forces shape the creation and maintenance of a group? According to the insight of Olson (1965), groups or organizations are characterized by the furtherance of the *common interests* of their members. However, "the achievement of any common goal or the satisfaction of any common interest means that a public or collective good has been provided for that group" (Olson 1965,

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I thank Carlos Pimienta for his suggestion and comments. The scholarship provided by Australian School of Business is acknowledged. The usual disclaimer applies.

p. 15). That is, it is the fundamental function of organizations to provide public goods. Though all of the members of a group have a common interest in achieving the collective benefit, they have no common interest in sharing the cost of providing that collective benefit. Therefore, *free-rider* problem will be prevalent in organizations if they fail to provide some *sanction* or some *attraction* distinct from the public good itself.

Olson (1965, Chapter I.F) (see also Olson 1982, Chapter 2) points out that groups are supported because they can find *selective incentives*. A selective incentive is "one that applies selectively to the individuals depending on whether they do or do not contribute to the provision of the collective good" (Olson, 1982, p. 21). Olson (1965) also points out that it is difficult for large groups to find selective incentives.

In this paper we follow Olson's logic of collective action. We show that competitive grouping based on individuals' group contributions increases cooperation and efficiency in an environment of voluntarily provision of public goods. In other words, *contribution-based grouping* provides a selective incentive which can be used to distinguish among individuals: an individual who does not contribute is ostracized, and an individual who contributes can be invited into a charmed group. This incentive results in efficient, or almost efficient, allocation. Precisely, if the players are grouped according to their contributions and if their endowments are heterogeneous, then there exist two positive contribution equilibria: one is fully efficient in the sense that all individuals contribute fully, and the other is near fully efficient in the sense that almost all individuals contribute fully.

Contribution-based grouping and social development appear to have gone hand in hand. Falling barriers to international trade and investment, and increased competition in product and service markets around the world make firms pay more attention to efficiency for success and even survival (Roberts, 2004, Chapter 5). With the resulting increase in competition, contribution-based grouping becomes more important. Moreover, North, Wallis and Weingast (2009) define modern societies by open accesses to political and economic opportunities. In those open access orders, access to organizations is an *impersonal* right that all citizens possess. For instance, North, Wallis and Weingast (2009) write:

> . . . impersonal categories of individuals, often called citizens, interact over wide areas of social behavior with no need to be cognizant of the individual identity of their partners. Identity, which in natural states is inherently personal, becomes defined as a set of impersonal characteristics in open access orders. The ability to form organizations that the larger society supports is open to everyone who meets a set of minimal and impersonal criteria (2009, p. 2).

Therefore, in modern open access orders individuals are identified and grouped according to their contribution rather than their privilege, race, or gender.

Our study provides a complement to the normative theory of public goods provision (Lindahl 1919, Samuelson 1954 and Foley 1970). While this normative theory characterizes efficiency in public goods economies clearly and rigorously, it leaves the question of *how* an economy may attain efficiency unanswered. This paper contributes to the literature on the positive theory of providing incentives associated with the free-rider problem (Clarke 1971, Groves 1973, Green and Laffont 1979, Bergstrom, Blume and Varian 1986, Laffont 1987, etc.)

Our model is similar to the *local public goods* model.[1] In the local public goods model, there are n individuals and a large number of locations in which they can choose to live. Our model differs from it by imposing the restriction that the accommodation capacity of each location is fixed, and each individual has to compete for a location.

The current article is related to Gunnthorsdottir, Vragov, Seifert and McCabe (2010) (henceforth GVSM). In GVSM, every individual has the same endowment and thus equal ability to make a contribution. Homogenous endowment is a very restrictive and unrealistic assumption. We depart from their paper by introducing unequal abilities to contribute. It turns out that this relaxation increases the difficulty of finding the equilibria of the game dramatically.

The paper is organized as follows. Section 2 introduces the model. We then finger out and characterize all the equilibria of the game in Section 3. Finally, Section 4 discusses the the limits of our model and possible extensions.

2. The Model

The set of players is $N := \{1, 2, \ldots, n\}$. Each player $i \in N$ has an endowment $w_i > 0$. The distribution of endowments is common knowledge. Each player $i \in N$ makes a contribution $s_i \in [0, w_i]$ to a public account, and keeps the remainder $(w_i - s_i)$ in her private account. The return from the private account is set to 1, the return from the public account is the *Marginal per Capita Return* (MRCP) $m \in (1/\varphi, 1)$. After their investment decisions, all players are ranked according to their public contributions and divided into G groups of equal size φ, so $G = n/\varphi$. Ties for group membership are broken at random. The φ players with the highest contributions are put into Group 1; then φ players with the next highest contributions are put into Group 2, and so on. Payoffs are computed after players have been grouped. Each player's payoff consists of the amount kept in her private account, plus the total public contribution of all players in the group she has been assigned to multiplied by the MPCR m.

[1]See Scotchmer (2002) and Ray (2007, Example 3.3.2(1), p. 28).

Given the other players' contributions profile $(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) = \boldsymbol{s}_{-i}$, let $U_i(s_i, \boldsymbol{s}_{-i})$ be player i's expected payoff from contributing s_i. Let $\mathbb{P}_{(s_i, \boldsymbol{s}_{-i})}(k)$ be i's probability of entering group k when the contribution profile is $(s_i, \boldsymbol{s}_{-i}) = \boldsymbol{s}$, where $k = 1, \ldots, G$; for simplicity we henceforth denote this probability by $\mathbb{P}_{s_i}(k)$. Let S^k_{-i} be the total contribution in group k except for player i. Therefore, player i's expected payoff $U_i(s_i, \boldsymbol{s}_{-i})$ from a contribution combination $\boldsymbol{s} = (s_i, \boldsymbol{s}_{-i})$ can be expressed as follows:

$$U_i(s_i, \boldsymbol{s}_{-i}) = (w_i - s_i) + \sum_{k=1}^{G} \mathbb{P}_{(s_i, \boldsymbol{s}_{-i})}(k) \cdot \left[m \cdot (S^k_{-i} + s_i) \right]. \tag{1}$$

The model can now been transformed into a normal form game. The set of players is N; each player i's strategy is her contribution s_i. Her strategy space is the interval $[0, w_i] \subseteq \mathbb{R}$; finally, player i's payoff function is defined by (1) for all $i \in N$. The Nash equilibrium is defined as follows:

Definition 1. A contribution profile $\boldsymbol{s} = (s_1, \ldots, s_n)$ is a *Nash equilibrium* if and only if $U_i(\boldsymbol{s}) \geqslant U_i(s'_i, \boldsymbol{s}_{-i})$ for all $s'_i \neq s_i$ and all $i \in N$.

For simplicity of expression, I now introduce two assumptions. The consequences when these assumptions fail will be discussed in Appendix B.

Assumption A *Each player's endowment is either* $w_i = H$ *or* $w_i = L < H$.

It is common to divide people of a society into two parts in the literature. For example, in Acemoglu and Robinson (2006) citizens consist of *poor* and *rich*. In Esteban and Ray (2011) there are two ethnic or religious affiliations (*Hindu* and *Muslim* in their model).[2] Finally, in the core model of Besley and Persson (2011) an *incumbent group* and an *opposition group* compete for political power and make decisions. The method developed in this paper can be extended to deal with the general case where there are more than two endowment levels with the expense of more complicated mathematical computation.

For what follows, we apply the following simplification without loss of generality: we normalize $L = 1$, and let $\Delta w = H - 1 > 0$ be the gap between the high endowment H and low endowment $L = 1$. We call a player with endowment H a "High", and a player with endowment 1 a "Low". Let N_H be the set of Highs and N_L the set of Lows. Their respective counts are $n_H := |N_H|$ and $n_L := |N_L|$. It follows that $N_H \cup N_L = N$, or equivalently, $n_H + n_L = n$. By the algorithm of integer division (Hardy and Wright, 2008), there exist some nonnegative integers A, B, h and ℓ, with $h < \varphi$ and $\ell < \varphi$, such that the counts of Highs and Lows can be expressed as:

$$\begin{cases} n_H = A\varphi + h, \\ n_L = B\varphi + \ell. \end{cases}$$

[2]Esteban and Ray (2008) divide a society into either economic groups (poor and rich) or ethnic groups.

Assumption B *The count of each type, High and Low, is more than, and not a multiple of the group size φ; that is,*

- $A \geqslant 1$, $B \geqslant 1$, *and* $A + B = G - 1$;
- $h \geqslant 1$, $\ell \geqslant 1$, *and* $h + \ell = \varphi$.

Roughly speaking, Assumption B just requires that the *minority* is not too weak. Formally, the number of individuals with the same endowments is larger than but not a multiple of the given group size. We relax this assumption in Appendix B.

We need to define one more basic concept, which will be crucial when we identify all the game's equilibria.

Definition 2. Let $C_r \subseteq N$. We call C_r a *class* if each player $i \in C_r$ contributes the same; that is, $i, j \in C_r$ if and only if $s_i = s_j$. We call a player $i \in C_r$ a *C_r-player*.

Given a contribution profile $\boldsymbol{s}$, the players can be divided into $R(\boldsymbol{s}) \leqslant n$ classes; we henceforth omit the argument $\boldsymbol{s}$. Let $\mathcal{C}$ be the family of all classes, i.e., $\mathcal{C} = \{C_1, \ldots, C_R\}$. Both $\mathcal{C}$ and $\{N_H, N_L\}$ partition N; that is, $\bigcup_{r=1}^{R} C_r = N_H \cup N_L = N$. In a class $C_r \in \mathcal{C}$, there are c_r players; the contribution of each player in C_r is s^r; that is, $|C_r| = c_r$, and $s_i = s^r$ for all $i \in C_r$. We index the classes such that $s^{r+1} < s^r$, where $r + 1 \leqslant R$; hence, C_1 is the class consisting of the highest contributors, and C_R is the class consisting of the lowest contributors. For each class C_r, we can find nonnegative integers D_r and $\widetilde{c}_r < \varphi$ such that the count of C_r-players can be expressed as

$$c_r = |C_r| = D_r \varphi + \widetilde{c}_r. \tag{2}$$

We now move on to solve the model. In the next section, we find and characterize all the Nash equilibria with positive individual contributions.

3. Equilibria

We begin with the following simple observation, which characterizes the most inefficient equilibrium.

Lemma 3. *Contributing nothing, i.e., $s_i = 0$ for all player $i \in N$ is a Nash equilibrium. This is the only equilibrium satisfying $|\mathcal{C}| = 1$.*

Proof. Let $s_j = 0$ for all players $j \neq i$. Player i obtains $(w_i - s_i) + m s_i = w_i - (1 - m) s_i$ if she contributes s_i. Her best response is therefore $s_i = 0$.

To verify that $s_i = 0$ for all player $i \in N$ when $|\mathcal{C}| = 1$, let $s^1 > 0$. Consider any player $i \in N$. She gets $(w_i - s^1) + m\varphi s^1$ if she contributes s^1, but if she deviates and contributes 0, she enters the last group G, and gets

$$\begin{aligned} w_i + m(\varphi - 1)s^1 &= (w_i - ms^1) + m\varphi s^1 \\ &> (w_i - s^1) + m\varphi s^1 \end{aligned}$$

since $m < 1$. Hence, $s_i = 0$ for each player $i \in N$ in an equilibrium with only one class. □

The equilibrium with $s_i = 0$ for all $i \in N$ always exists as long as m, the MPCR, is less than 1, but it is not a dominant strategy equilibrium. Since $s_i = 0$ for all $i \in N$ if $|\mathcal{C}| = 1$ by Lemma 3, in any equilibrium with positive contributions it must be that $|\mathcal{C}| \geqslant 2$. Theorem 4 here below will show that there are only two (pure strategy) Nash equilibria involving positive contributions: a fully efficient equilibrium (FEE), and a near-efficient equilibrium (NEE). These two equilibria can be characterised as follows:

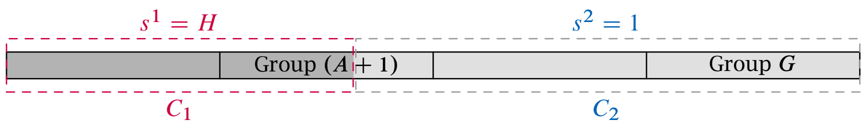
FEE. There are two classes: C_1 is identical to N_H, C_2 is identical to N_L, and all players contribute fully; that is,

- *Classes*: $|\mathcal{C}| = 2$, where $C_1 = N_H$ and $C_2 = N_L$.
- *Strategies*: $s_i = \begin{cases} H & \text{if } i \in C_1 \\ 1 & \text{if } i \in C_2. \end{cases}$

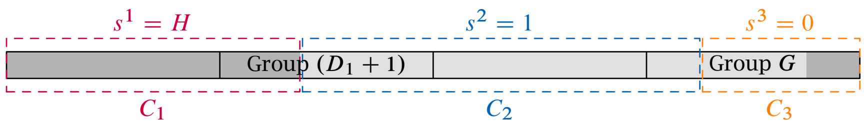
NEE. There are three classes: C_1 consists of Highs, C_2 consists of Lows, and C_3 consists of the players who are not in C_1 or C_2. Both C_1 and C_2-players contribute fully, but C_3-players contribute nothing. The sum of C_2 and C_3-players together is greater than and not a multiple of group size; the count of C_3-players is less than the group size; that is,

- *Classes*: $|\mathcal{C}| = 3$, where $\begin{cases} C_1 \subseteq N_H, c_1 > \varphi \text{ and } \tilde{c}_1 > 0 \\ C_2 \subseteq N_L, c_2 + c_3 > \varphi, \text{ and } \tilde{c}_2 + \tilde{c}_3 \neq \varphi \\ C_3 = N \setminus (C_1 \cup C_2) \text{ and } c_3 < \varphi. \end{cases}$
- *Strategies*: $s_i = \begin{cases} H & \text{if } i \in C_1 \\ 1 & \text{if } i \in C_2 \\ 0 & \text{if } i \in C_3. \end{cases}$

In both equilibria with positive contributions, strategies only take one of three forms: full contribution of the high endowment (H), full contribution of the low endowment (1) or zero contribution. Figure 1(a) and (b) illustrate FEE and NEE, respectively, where the dark gray sections represent Highs, the light gray sections Lows. The players' strategies s_i are shown on top of the horizontal bars. The segments in the bars represent groups. For illustration purposes and without loss of generality, only four groups are shown.



(a) FEE



(b) NEE

FIGURE 1. The two equilibrium configurations with positive contributions (light gray sections are Lows, dark gray sections are Highs).

Theorem 4. *If there is an equilibrium with positive contributions, then it is a FEE or NEE.*

The proof of Theorem 4 is presented in Appendix A. Here are some examples of the application of this theorem.

Example 5. Let $n = 12$, $n_H = n_L = 6$, $\varphi = 4$, $L = 1$ and $H = 1.5$. According to Theorem 4, we only need to consider FEE and NEE:

There is no FEE since any player $i \in C_2$ has an incentive to reduce her contribution: If i contributes 1, she enters the second group with probability $2/6$, and the third group with probability $4/6$, so the expected payoff is $0.5 \times (2/6 \times 5 + 4/6 \times 4) = 13/6$, but if she contributes 0, she enters the third group with certainty and obtains $1 + 0.5 \times 3 = 5/2 > 13/6$.

Hence, if there exists an equilibrium with positive contributions, it must be a NEE. As the following table shows, the unique equilibrium with positive contributions is $\left(\langle 1.5, 1.5, 1.5, 1.5\rangle, \langle 1.5, 1.5, 1, 1\rangle, \langle 1, 1, 0, 0\rangle\right)$.

c_1	c_2	c_3	NEE?	Deviator	Deviation ($s_i \to s'_i$)
5	6	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
5	5	2	No	$i \in C_3 \cap N_H$	$0 \to 1 + \varepsilon$
5	4	3	No	$i \in C_3 \cap N_H$	$0 \to 1 + \varepsilon$
6	5	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
6	4	2	Yes	$\varnothing$	
6	3	3	No	$i \in C_3 \cap N_L$	$0 \to 1$

Example 6. In a game with parameters as in Example 5, now let $n_H = 7$ instead of previously 6. It can be verified that there is no FEE. By Theorem 4, it suffices to show that there is no NEE either. There are eight cases to consider:

c_1	c_2	c_3	NEE?	Deviator	Deviation ($s_i \to s'_i$)
5	5	2	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
5	4	3	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
6	5	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
6	4	2	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
6	3	3	No	$i \in C_3 \cap N_H$	$0 \to 1+\varepsilon$
7	4	1	No	$i \in C_2 \subseteq N_L$	$1 \to 0$
7	3	2	No	$i \in C_1 = N_H$	$H \to 1+\varepsilon$
7	2	3	No	$i \in C_1 = N_H$	$H \to 1+\varepsilon$

Example 7. This example relies on some results in Appendix A. The general method developed so far can be used to reprove Observation 2 in GVSM (2010). GVSM's parameter z corresponds to $c_R = |C_R|$, the number of players in the last class. If $H = L = 1$ and if there exists an equilibrium with positive contributions, it can be characterized as follows:

$$|\mathcal{C}| = 2, \quad s^1 = 1, \quad s^2 = 0, \quad \text{and} \quad c_2 < \varphi.$$

By Lemma A.1(a) (in Appendix A), in any equilibrium with positive contributions $c_1 > 0$, $\tilde{c}_1 > \varphi$, and $s^1 = 1$. Now consider the last class C_R:

- If $c_R > \varphi$ and $\tilde{c}_R > 0$ in equilibrium, then $s^R = 1$ by Claim 1 in Appendix A. However, this means that $|\mathcal{C}| = 1$ and $\tilde{c}_1 = 0$, a contradiction to Lemma A.1(a).

- Assume $\tilde{c}_R = 0$ in equilibrium. Then $s^2 = 0$ by Lemma A.1 (e). By the same logic as in Lemma A.1 (c), there cannot exist a class C_r satisfying $0 < s^r < 1$; hence, $|\mathcal{C}| = 2$. According to Lemma A.1(a) $\tilde{c}_1 > 0$. If $\tilde{c}_2$ were zero, it would contradict our initial assumption at the beginning of Section 3.1 that the total number of players $n = G\varphi$.

- Thus, it must be that $c_R < \varphi$. It follows that $s^R = 0$ by Lemma A.1 (e). An argument analogous to Lemma A.1 (c) shows that $|\mathcal{C}| = 2$.

3.1. *Existence of a fully efficient equilibrium (FEE)*

A FEE exists if and only if

- Player $i \in C_2$ has no incentive to reduce her contribution from 1 to 0, and

- Player $i \in C_1$ has no incentive to reduce her contribution from H to $1+\varepsilon$, 1, or 0, where $\varepsilon \in \mathbb{R}_{++}$.

We first consider C_2, then C_1. We use $U_{s_i}^{w_i}(C_r)$ to denote player i's expected payoff when her endowment is $w_i \in \{H, 1\}$, she contributes $s_i \in [0, w_i]$, and is in class C_r. We develop our analysis with the help of Figure 2.

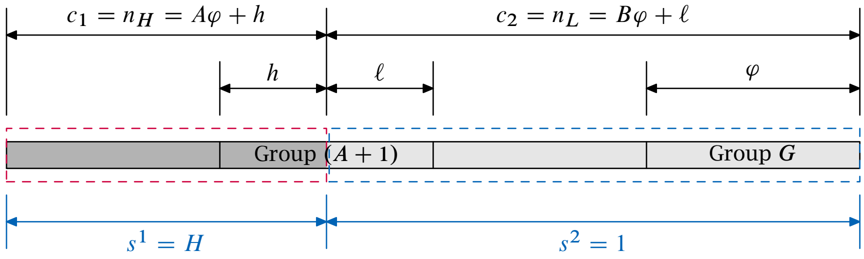


FIGURE 2. The distribution of players in a FEE

Proposition 8. *Let $M = (1-m)/m$. A FEE exists if and only if*

$$\frac{Mn_L}{\Delta w \cdot \ell} \leqslant h \leqslant \min\left\{\frac{[(\varphi-1)\Delta w - MH]n_H}{\Delta w \cdot \ell}, \frac{(\ell - M)n_H}{\ell}\right\}. \tag{3}$$

The remainder of this subsection will be devoted to the proof of Proposition 8.

Incentives to Deviate for C_2-Players in a FEE. Fix the contribution profile $\boldsymbol{s}_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$ satisfying $s_j = w_j$ for all $j \in N \setminus \{i\}$. For any player $i \in C_2 = N_L$, if she contributes 1, she enters the following groups with positive probabilities: $A+1, A+2, \ldots, G$ (see Figure 2). The probabilities are:

$$\mathbb{P}_1(k) = \begin{cases} \ell/n_L & \text{if } k = A+1 \\ \varphi/n_L & \text{if } k = A+2, \ldots, G. \end{cases}$$

Since $\sum_{k=A+1}^{G} \mathbb{P}_1(k) = 1$, we have $\sum_{k=A+2}^{G} \mathbb{P}_1(k) = 1 - \mathbb{P}_1(A+1) = 1 - \ell/n_L$. For ease of expression, let

$$S^{A+1} = hH + \ell;$$

that is, S^{A+1} is the sum of contributions in Group $(A+1)$ from the full contribution profile $\boldsymbol{s} = (s_i = 1, \boldsymbol{s}_{-i})$. By (1), player i's expected payoff from contributing $s_i = 1$ is

$$\begin{aligned} U_1^L(C_2) &= (w_i - s_i) + m\left[\mathbb{P}_1(A+1)S^{A+1} + \sum_{k=A+2}^{G} \mathbb{P}_1(k)\varphi\right] \\ &= (1-1) + m\left[\mathbb{P}_1(A+1)S^{A+1} + \sum_{k=A+2}^{G} \mathbb{P}_1(k)\varphi\right] \\ &= m\left[\frac{\ell}{n_L}S^{A+1} + \left(1 - \frac{\ell}{n_L}\right)\varphi\right] \\ &= m\left(\varphi + \frac{h\ell\Delta w}{n_L}\right), \end{aligned}$$

where the last equality holds since $S^{A+1} - \varphi = (hH + \ell) - (h + \ell) = h(H-1) = h\Delta w$.

If player $i \in C_2$ deviates and contributes $s_i < 1$, she enters group G, and her payoff is

$$(1 - s_i) + m\left[(\varphi - 1) + s_i\right] = 1 + m(\varphi - 1) - (1 - m)s_i;$$

hence, the optimal deviation is $s_i = 0$ since $1 - m > 0$ with payoff is $U_0^L(C_2) = 1 + m(\varphi - 1)$.

Hence, player $i \in C_2$ has no incentive to reduce her contribution from 1 to 0 if and only if $U_1^L(C_2) \geqslant U_0^L(C_2)$; that is,

$$h \geqslant \frac{(1 - m)n_L}{m\ell \cdot \Delta w} = \frac{Mn_L}{\ell \cdot \Delta w}, \tag{4}$$

where $M = (1 - m)/m$. Because $m \in \big(1/\varphi, 1\big)$, we know that $M \in (0, \varphi - 1)$.

Incentives to Deviate for C_1-Players in a FEE. Since we now consider a player $i \in C_1 = N_H$, we rewrite the full contribution profile as $\boldsymbol{s} = (s_i = H, \boldsymbol{s}_{-i})$, where $s_j = w_j$ for any $j \in N \setminus \{i\}$. If player $i \in C_1$ contributes $s_i = H$, she enters Group $1, 2, \ldots, A, A + 1$ with positive probabilities, which are

$$\mathbb{P}_H(k) = \begin{cases} \varphi/n_H & \text{if } k = 1, \ldots, A \\ h/n_H & \text{if } k = A + 1. \end{cases}$$

Hence, i's expected payoff from contributing $s_i = H$ is

$$\begin{aligned} U_H^H(C_1) &= (H - H) + m\left[\sum_{k=1}^{A} \mathbb{P}_H(k)\varphi H + \mathbb{P}_H(A + 1)S^{A+1}\right] \\ &\overset{\langle 1\rangle}{=} m\left[\left(1 - \frac{h}{n_H}\right)\varphi H + \frac{h}{n_H}S^{A+1}\right] \\ &\overset{\langle 2\rangle}{=} m\left(\varphi H - \frac{h\ell\Delta w}{n_H}\right), \end{aligned}$$

where $\langle 1\rangle$ holds because $\sum_{k=1}^{A} \mathbb{P}_H(k) = 1 - \mathbb{P}_H(A+1) = 1 - h/n_H$, and $\langle 2\rangle$ holds because $\varphi H - S^{A+1} = \varphi H - (hH + \ell) = \ell H - \ell = \ell\Delta w$.

If player $i \in C_1$ contributes $s_i \in (1, H)$, she enters group $(A+1)$ with certainty and obtains

$$\begin{aligned} U_{s_i}^H(C_1) &= (H - s_i) + m\left[(h - 1)\, H + \ell + s_i\right] \\ &= H + m\left[(h - 1)\, H + \ell\right] - (1 - m)s_i. \end{aligned} \tag{5}$$

From (5) we know that the optimal deviation is $s_i = (1 + \varepsilon) \to 1$ if player $i \in C_1$ wants to contribute $s_i \in (1, H)$. Thus,

$$\begin{aligned} \lim_{\varepsilon \to 0} U_{1+\varepsilon}^H(C_1) &= \lim_{\varepsilon \to 0}\Big\{H + m\left[(h - 1)\, H + \ell\right] - (1 - m)\,(1 + \varepsilon)\Big\} \\ &= H + m(S^{A+1} - H) - (1 - m) \\ &= mS^{A+1} + (1 - m)\Delta w. \end{aligned}$$

Hence, player $i \in C_1$ has no incentive to reduce her contribution from H to $1+\varepsilon$ if and only if $U_H^H(C_1) \geqslant \lim_{\varepsilon\to 0} U_{1+\varepsilon}^H(C_1)$; that is

$$h \leqslant n_H\left(1-\frac{M}{\ell}\right). \tag{6}$$

Note that (6) is *independent* of H or Δw: it is fully determined by the distribution of player types and the MPCR m.

Lemma 9 here below indicates that we do not need to consider whether $i \in C_1$ has an incentive to contribute 1 if she has no incentive to contribute $1+\varepsilon$.

Lemma 9. *If a player $i \in C_1$ has no incentive to reduce her contribution from H to $1+\varepsilon$, she also has no incentive to reduce her contribution from H to 1.*

Proof. If player $i \in C_1$ contributes 1, she enters Group $A+1, A+2, \ldots, G$ with positive probabilities. Therefore, her expected payoff is

$$\begin{aligned}
U_1^H(C_1) &= (H-1) + m\left[\mathbb{P}_1(A+1)\left[(h-1)H + \ell + 1\right] + \sum_{k=A+2}^{G} \mathbb{P}_1(k)\varphi\right] \\
&= \Delta w + m\left[\mathbb{P}_1(A+1)(S^{A+1} - \Delta w) + \varphi \sum_{k=A+2}^{G} \mathbb{P}_1(k)\right] \\
&\leqslant \Delta w + m\left[\mathbb{P}_1(A+1)(S^{A+1} - \Delta w) + (S^{A+1} - \Delta w)(1 - \mathbb{P}_1(A+1))\right] \\
&= mS^{A+1} + (1-m)\Delta w \\
&= \lim_{\varepsilon\to 0} U_{1+\varepsilon}^H(C_1),
\end{aligned}$$

where the inequality holds since

$$\begin{aligned}
S^{A+1} - \Delta w = (hH + \ell) - (H-1) &= [hH + (\varphi - h)] - H + 1 \\
&\geqslant (H + \varphi - 1) - H + 1 \\
&= \varphi.
\end{aligned}$$

Thus, $U_H^H(C_1) \geqslant U_1^H(C_1)$ when $U_H^H(C_1) \geqslant \lim_{\varepsilon\to 0} U_{1+\varepsilon}^H(C_1)$. □

Finally, if player $i \in C_1$ wants to contribute $s_i < 1$, she should contribute $s_i = 0$, so that her payoff is $U_0^H(C_1) = H + m(\varphi - 1)$. Hence, she has no incentive to contribute 0 if and only if $U_H^H(C_1) \geqslant U_0^H(C_1)$; that is,

$$h \leqslant \frac{[(\varphi-1)\Delta w - MH]n_H}{\Delta w \cdot \ell}. \tag{7}$$

Combining (4), (6) and (7), one obtains Proposition 8.

Comparative Statics of the FEE and Two Examples. It can be seen from (3) that when m is large enough, the FEE is an equilibrium for all possible parameters of the game. To illustrate, consider the extreme case: Let $m \to 1$, then $\lim_{m\to 1} M = \lim_{m\to 1}(1-m)/m = 0$. Then the left-hand side (LHS) of (3) approaches 0, the right-hand side (RHS) of (3) becomes

$$\min\left\{\frac{(\varphi-1)n_H}{\ell}, n_H\right\} = n_H,$$

and $0 \leqslant h \leqslant n_H$ always holds. This result is intuitive: $m \to 1$ means that if a player puts one dollar into the public account, her strategic risk becomes negligible.

In a FEE, the gap between Highs and Lows, Δw, cannot be very small. This result might strike the reader as counterintuitive since it implies that equality (in w_i) prevents a fully efficient solution. Consider once again the extreme case. Fixed all other parameters and let $\Delta w \to 0$, then

$$\lim_{\Delta w\to 0} \frac{Mn_L}{\Delta w \cdot \ell} = +\infty > h$$

so that (3) is violated. This result corresponds to GVSM (2009): when all players have the same endowment, it is not an equilibrium that all contribute fully.

Although a large enough Δw, or H, is a *necessary* condition for the existence of a FEE, it is not *sufficient*. To see this, let $H \to +\infty$, so that $\Delta w \to +\infty$, too; then (3) becomes

$$(3') \qquad 0 \leqslant h \leqslant \min\left\{\frac{(\varphi-1-M)n_H}{\ell}, \frac{(\ell-M)n_H}{\ell}\right\} = \frac{(\ell-M)n_H}{\ell}.$$

We can see that there exist ℓ and M such that (3′) fails. In particular, if $M \to \varphi-1$, or equivalently $m \to 1/\varphi$, then there is clearly no FEE no matter how high H is and no matter what the distribution of types is, since $\ell \leqslant \varphi - 1$.

Example 10. Let $m = 0.5$ [so $M = (1-m)/m = 1$], $\varphi = 4$, $n = 24$, $H = 3$. We refer to Figure 3. In the figure, each point n_H on the horizontal axis determines a particular ℓ according to the equation $n_L = n - n_H = B\varphi + \ell$, and such an ℓ determines: (a) the h by the equation $h = \varphi - \ell$, (b) the (3)-LHS, and (c) the (3)-RHS. Thus, if there is a h determined by a n_H that lies between the two curves, then there exists a FEE by Proposition 8.

Figure 3 indicates that there is a FEE if and only if $n_H = 18$. Note that $n_H = 4A + h$ yields $h = \ell = 2$ [the *red* point in the figure]; furthermore, $n_L = n - n_H = 6$, (3)-LHS $= 1.5$, and

$$(3)\text{-RHS} = \min\left\{\frac{(3\times 2-3)\times 18}{2\times 2}, \frac{(2-1)\times 18}{2}\right\} = 9;$$

thus, $1.5 < h = 2 < 9$; that is, (3) holds. We now show it is indeed an equilibrium:

In equilibrium, $i \in C_2$ gets $0.5 \times (2/6 \times 8 + 4/6 \times 4) = 2.7$. If she contributes 0, she gets $1 + 0.5 \times 3 = 2.5 < 2.7$. Hence, $i \in C_2$ has no incentive to deviate.

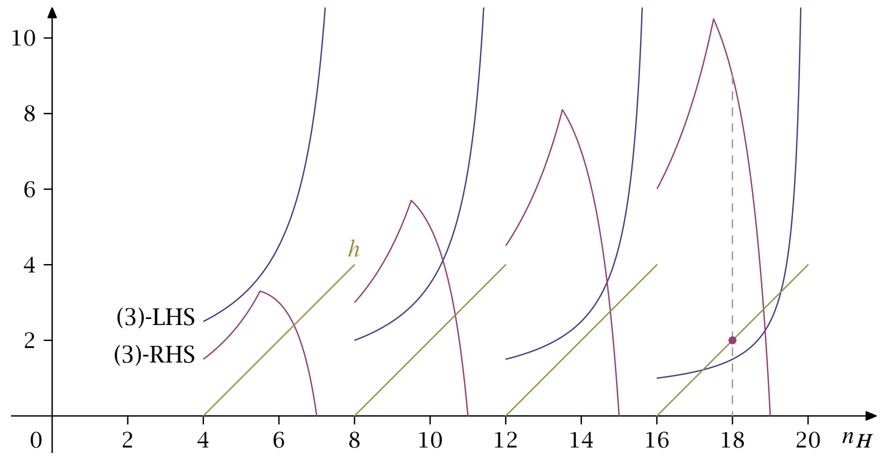


FIGURE 3. An example of FEE

In equilibrium, $i \in C_1$ gets $0.5 \times (16/18 \times 12 + 2/18 \times 8) = 5.8$; If she contributes $1 + \varepsilon$, she gets no more than $0.5 \times 8 + (1 - 0.5) \times 2 = 5$, which is less than 5.8; finally, if she contributes 0, she gets $3 + 0.5 \times 3 = 4.5 < 5.8$. Hence, $i \in C_1$ also has no incentive to deviate.

Example 11. In a game with parameters as in Example 5, now let H be unspecified. We want to find an H such that there exists a FEE. According to (3), H has to satisfy $h = 2 \geqslant 6/(2(H-1))$, which solves for $H \geqslant 2.5$. Because (3)-RHS holds when $H \geqslant 2.5$, this concludes the calculation.

3.2. *Existence of a near-efficient equilibrium (NEE)*

The NEE exists if and only if

- player $i \in C_3 \cap N_L$ has no incentive to increase her contribution from 0 to 1.
- player $i \in C_3 \cap N_H$ has no incentive to increase her contribution from 0 to $1 + \varepsilon$ or H.
- player $i \in C_2 \cap N_L$ has no incentive to reduce her contribution from 1 to 0.
- player $i \in C_1 \cap N_H$ has no incentive to reduce her contribution from H to $1 + \varepsilon$ or 0.

Since Example 5 already showed that this equilibrium is possible in some cases, there is no existence problem. We provide here a general overview of the conditions under which it exists.

Let c_3^H be the count of Highs in C_3, and c_3^L be the count of Lows in C_3. Then $c_3 = c_3^H + c_3^L < \varphi$ and $c_3^H \neq h$, otherwise $\tilde{c}_1 = 0$, which contradicts

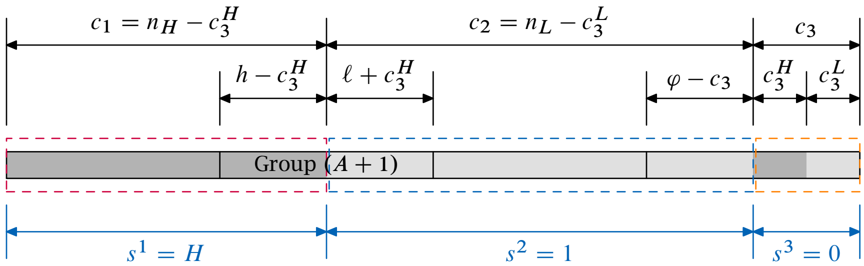


FIGURE 4. The distribution of players in a NEE

Lemma A.1(a). We have

$$c_1 = n_H - c_3^H = \begin{cases} A\varphi + h - c_3^H & \text{if } c_3^H < h \\ (A-1)\varphi + h + (\varphi - c_3^H) & \text{if } c_3^H > h, \end{cases} \tag{8}$$

and

$$c_2 = n_L - c_3^L = \begin{cases} B\varphi + \ell - c_3^L & \text{if } c_3^L \leqslant \ell \\ (B-1)\varphi + \ell + (n - c_3^L) & \text{if } c_3^L > \ell. \end{cases} \tag{9}$$

It is obviously impossible that $c_3^H > h$ and $c_3^L > \ell$ hold simultaneously since $h + \ell = \varphi$. It also can be seen from (8) and (9) that there are three situations to consider: (1) $c_3^H < h$ and $c_3^L \leqslant \ell$, (2) $c_3^H < h$ and $c_3^L > \ell$, and (3) $c_3^H > h$ and $c_3^L \leqslant \ell$. In this paper we only analyze the simplest case, in category (1):

$$c_3^H < h, \quad c_3^L < \ell, \quad \text{and} \quad c_3^H + c_3^L < \varphi.$$

The other cases can be analyzed in the same manner. We develop our analysis with the help of Figure 4, which illustrates the distribution of players in a NEE.

Incentives to Deviate for C_3-Players in a NEE. Firstly, for player $i \in C_3 \cap N_L$, her payoff from contributing 0 is

$$U_0^L(C_3) = 1 + m(\varphi - c_3). \tag{10}$$

If she contributes 1, then there are $c_2 + 1$ players contributing 1 and player i enters Group $A+1, \ldots, G$ with positive probabilities, which are

$$\mathbb{P}_1(k) = \begin{cases} (\ell + c_3^H)/(c_2 + 1) & \text{if } k = A + 1 \\ \varphi/(c_2 + 1) & \text{if } k = A + 2, \ldots, G - 1 \\ (\varphi - c_3 + 1)/(c_2 + 1) & \text{if } k = G. \end{cases}$$

Let $S = (h - c_3^H)H + (\ell + c_3^H)$. Thus, player i's expected payoff from contributing 1 is

$$\begin{aligned} U_1^L(C_3) &= m\left[\mathbb{P}_1(A+1)S + \sum_{k=A+2}^{G-1}\mathbb{P}_1(k)\varphi + \mathbb{P}_1(G)(\varphi - c_3 + 1)\right] \\ &= \frac{m}{c_2+1}\left[(\ell + c_3^H)S + (n_L - \varphi - \ell)\varphi + (\varphi - c_3 + 1)^2\right], \end{aligned} \tag{11}$$

where the last equality holds because

$$\sum_{k=A+1}^{G-1}\mathbb{P}_1(k) = 1 - \frac{\ell + c_3^H}{c_2+1} - \frac{\varphi - c_3 + 1}{c_2+1} = \frac{(c_2 + c_3^L) - \varphi - \ell}{c_2+1} = \frac{n_L - \varphi - \ell}{c_2+1}.$$

Hence, player $i \in C_3 \cap N_L$ has no incentive to deviate from contributing 0 to contributing 1 if and only if $U_0^L(C_3) \geqslant U_1^L(C_3)$.

Secondly, for $i \in C_3 \cap N_H$, her payoff from contributing $s_i = H$ is

$$U_0^H(C_3) = H + m(\varphi - c_3). \tag{12}$$

If player i contributes $1+\varepsilon$, she enters group $(A+1)$ and obtains

$$\begin{aligned} \lim_{\varepsilon\to 0} U_{1+\varepsilon}^H(C_3) &= \lim_{\varepsilon\to 0}\left\{H - 1 - \varepsilon + m\left[(h - c_3^H)H + (\ell + c_3^H - 1) + 1 + \varepsilon\right]\right\} \\ &= \Delta w + mS. \end{aligned} \tag{13}$$

If player i contributes H, then there are $c_1 + 1$ players contributing H; player i enters Group $1, \dots, A+1$ with positive probabilities, which are

$$\mathbb{P}_H(k) = \begin{cases} \varphi/(c_1+1) & \text{if } k = 1, \dots, A \\ (h - c_3^H + 1)/(c_1+1) & \text{if } k = A+1. \end{cases}$$

Thus, player i's expected payoff is

$$\begin{aligned} U_H^H(C_3) &= m\left[\sum_{k=1}^{A}\mathbb{P}_H(k)\varphi H + \mathbb{P}_H(A+1)((h - c_3^H + 1)H + \ell + c_3^H - 1)\right] \\ &= m\left[\left(1 - \frac{h - c_3^H + 1}{c_1+1}\right)\varphi H + \frac{h - c_3^H + 1}{c_1+1}(S + \Delta w)\right] \\ &= \left(\frac{m}{c_1+1}\right)\left[(n_H - h)\,\varphi H + (h - c_3^H + 1)\,(S + \Delta w)\right]. \end{aligned} \tag{14}$$

Hence, player $i \in C_3$ has no incentive to deviate if and only if the following conditions are satisfied:

$$\begin{cases} (10) \geqslant (11): & i \in C_3 \cap N_L \text{ has no incentive to deviate from 0 to 1} \\ (12) \geqslant (13): & i \in C_3 \cap N_H \text{ has no incentive to deviate from 0 to } 1+\varepsilon \\ (12) \geqslant (14): & i \in C_3 \cap N_H \text{ has no incentive to deviate from 0 to } H. \end{cases} \tag{IC$_3$}$$

Incentives to Deviate for C_2-Players in a NEE. Recall that C_2 consists of Lows. If $i \in C_2 \subseteq N_L$ contributes 1, she gets

$$U_1^L(C_2) = \frac{m}{c_2}\left[(\ell + c_3^H)S + (c_2 - \ell - c_3^H - \varphi + c_3)\varphi + (\varphi - c_3)^2\right]; \tag{15}$$

if she contributes 0, she gets

$$U_0^L(C_2) = 1 + m(\varphi - c_3 - 1). \tag{16}$$

Thus, $i \in C_2 \cap N_L$ has no incentive to deviate if and only if

$$(15) \geqslant (16): \quad i \in C_2 \subseteq N_L \text{ has no incentive to deviate from 1 to 0.} \tag{IC$_2$}$$

Incentives to Deviate for C_1-Players in a NEE. C_1 consists of Highs. For $i \in C_1 \subseteq N_H$, if she contributes H, her expected payoff is

$$U_H^H(C_1) = m\left[\left(1 - \frac{h - c_3^H}{c_1}\right)\varphi H + \frac{h - c_3^H}{c_1} S\right]. \tag{17}$$

If she contributes $1 + \varepsilon$, she obtains

$$\begin{aligned} \lim_{\varepsilon \to 0} U_{1+\varepsilon}^H(C_1) &= \lim_{\varepsilon \to 0}\left\{H - 1 - \varepsilon + m\left[(h - c_3^H - 1)H + \ell + c_3^H + 1 + \varepsilon\right]\right\} \\ &= mS + (1 - m)\Delta w. \end{aligned} \tag{18}$$

A similar argument as in Lemma 9 shows that we need not consider whether $i \in C_1 \cap N_H$ has any incentive to contribute 1 if she has no incentive to contribute $1+\varepsilon$. We can therefore immediately consider the last possible deviation. If player i contributes 0, she obtains

$$U_0^H(C_1) = H + m(\varphi - c_3 - 1). \tag{19}$$

Thus, $i \in C_1 \subseteq N_H$ has no incentive to deviate if and only if

$$\begin{cases} (17) \geqslant (18): & i \text{ has no incentive to deviate from } H \text{ to } 1 + \varepsilon \\ (17) \geqslant (19): & i \text{ has no incentive to deviate from } H \text{ to } 0. \end{cases} \tag{IC$_1$}$$

Proposition 12 summarize this section's findings:

Proposition 12. *The NEE exists if and only if* (IC$_3$), (IC$_2$), *and* (IC$_1$) *are all satisfied.*

3.3. *Coexistence of NEE and FEE*

By Theorem 4 we know that if there are equilibria with positive contributions, it is a FEE or NEE. Can these two equilibria ever coexist? We will now show with an example that this is possible. Example 5 demonstrated that this game has a NEE. Example 11 showed that the game has a FEE if and only if $H \geqslant 2.5$.

We now show that if $H = 2.5$ there exists, in addition to the FEE, the following NEE:

$$\left(\langle H, H, H, H\rangle, \langle H, H, 1, 1\rangle, \langle 1, 1, 1, 0\rangle\right).$$

- For player $i \in C_3 \subseteq N_L$, her equilibrium payoff is $U_0^L(C_3) = 1 + 3/2 = 5/2$; if she contributes 1, the expected payoff is $U_1^L(C_3) = 1/2 \times (2S/6 + 4/6 \times 4) = 5/2 = U_0^L(C_3)$.

- For every player $i \in C_2 \subseteq N_L$, her equilibrium payoff is $U_1^L(C_2) = 1/2 \times (2/5 \times 7 + 3/5 \times 3) = 2.3$; if she contributes 0, the payoff is $U_0^L(C_2) = 1 + 1/2 \times 2 = 2 < U_1^L(C_2)$.

- Finally, for player $i \in C_1 = N_H$, she gets $U_H^H(C_1) = 1/2 \times (4/6 \times 4H + 2S/6) = 4.5$ in equilibrium; if she contributes $1 + \varepsilon$, the payoff is $\lim_{\varepsilon \to 0} U_{1+\varepsilon}^H(C_1) = S/2 + (H - 1)/2 = 4.25 < U_H^H(C_1)$; if she contributes 0, the payoff is $U_0^H(C_1) = H + 2/2 = 3.5 < U_H^H(C_1)$.

Note however that the unique equilibrium with positive contributions is the FEE if $H > 2.5$: Since it is required that $c_1 > 4$ and $\tilde{c}_1 > 0$ in any equilibrium with positive contributions, c_3^H can only take two possible values: either $c_3^H = 1$ or $c_3^H = 0$. However, $c_3^H = 1$ is impossible. This is because if a High has no incentives to contribute 0 in the FEE, she also has no incentive to contribute 0 when there is at least one Low in Group G contributing 0. Hence, we only need to consider the case of $c_3^H = 0$. By (10),

$$U_0^L(C_3) = 1 + \frac{4 - c_3}{2} = \frac{6 - c_3}{2}. \tag{10'}$$

By (11),

$$U_1^L(C_3) = \frac{4H + 4 + (5 - c_3)^2}{14 - 2c_3}, \tag{11'}$$

where $c_3 = 1, 2, 3$. Then

$$(11') - (10') = \frac{3c_3 + 4H - 13}{14 - 2c_3} > \frac{3(c_3 - 1)}{14 - 2c_3} > 0,$$

for any $c_3 = 1, 2, 3$, which means that $U_0^L(C_3) < U_1^L(C_3)$; that is, any C_3-player will deviate no matter how many players contribute 0 in Group G. We thus proved that no player will contribute 0 if $H > 2.5$, in other words, the FEE is the unique equilibrium with positive contributions if $H > 2.5$.

4. Discussion and Extensions

There are a number of questions regarding our stylized model. In this section we outline some of these questions to expand its realism and relevance.

Voluntary Participation. In this paper we assume that all individuals participate the competition voluntarily, so there are no individual rationality constraints. However, as Dixit and Olson (2000) show, participation decision may destroy efficiency. In next study, it would be important to check whether our main results are immune to individuals' participation consideration.

Coalition Formation. In our model, every individual makes her contribution decision non-cooperatively. However, *coalitions* may arise. Ray and Vohra (2001) (also see Ray 2007, Section 6.2) consider a model of public goods provision when all agents can write binding agreements. They find that coalition formation is a potential source of inefficiency.[3] It is interesting to check whether our results are robust with respect to coalition formation when the *coalition structure*[4] is given.

Externalities among Groups. In our model there is no influence among groups once all individuals are grouped. Nevertheless, this is probably a restrictive requirement. As Esteban and Ray (2001) and Grossman and Helpman (2001) show, externalities among groups could be significant. Therefore, it is interesting to see how the inter-group actions affect our results.

Measurement and Enforcement Problem. A presumption of our model is that every individual's contribution can be identified precisely. This is a demanding requirement under some environments, especially when the number of interacting individuals is large. More importantly, while our mechanism is third-party-enforceable, it is unclear whether it is self-enforceable.[5]

Group Size and the Degree of Cooperation. The *Olson paradox* (Olson, 1965) says that smaller groups are more successful than larger groups in obtaining an optimal supply of collective actions. Esteban and Ray (2001) overturn the Olson paradox by considering a model with the following features: explicit intergroup interaction, collective prizes with a mix of public and private characteristics, and nonlinear lobbying costs. However, our model predicts that the degree of cooperation not only depends on the group size, but also the distribution of endowments in society (see Proposition 8). Because we assume only two endowment levels in the current paper, it is important to deplore further that how the distribution of endowments and group size influence collective actions.

[3] See Chatterjee, Dutta, Ray and Sengupta (1993), Ray and Vohra (1997), Ray and Vohra (1999), Maskin (2003), Hyndman and Ray (2007) and Ray (2007) for coalition formation.

[4] Given the set of players N, a *coalition structure* is just a partition of N. Hence, the G groups in our model is a potential coalition structure.

[5] See Barzel (2002) for a detailed discussion regarding the enforcement problem.

Incomplete Information. We assume complete information in our model. Potentially, the following two informational issues will affect our outcomes. (1) An individual may be uncertain about the others' endowments. (2) Individuals may uncertain about the number of potential participants. The next step is to relax the complete information assumption.

Appendix A. Proof of Theorem 4

The proof of Theorem 4 relies upon the five auxiliary results summarized in the following lemma:

Lemma A.1. *If an equilibrium with positive contributions exists, it has the following properties:*

1. *The count of C_1-players is larger than and not a multiple of group size φ, and each C_1-player contributes fully. Formally, $c_1 > \varphi$, $\tilde{c}_1 > 0$, and $s_i = w_i$ if $i \in C_1$.*
2. *C_1 consists of Highs only; that is, $C_1 \subseteq N_H$.*
3. *There is no class C_r satisfying $1 < s^r < H$.*
4. *If the equilibrium consists of only two classes, it is a FEE.*
5. *If the count of C_R-players is less than or a multiple of the group size, then each C_R-player contributes nothing. Formally, if $c_R < \varphi$ or $\tilde{c}_R = 0$, then $s^R = 0$.*

Proof. We prove this lemma step by step.

(1). If $\tilde{c}_1 = 0$, then $c_1 = |C_1| = D_1 \cdot \varphi$ by (2). Consider any player $i \in C_1$. If $s_i = s^1$, she is always grouped with $(\varphi - 1)$ players contributing s^1 and gets $\left(w_i - s^1 + m\varphi s^1\right)$; if she contributes $s'_i = s^1 - \varepsilon > s^2$ where $\varepsilon \in \mathbb{R}$, she is in Group D_1 but is still grouped with $(\varphi - 1)$ players contributing s^1, and gets

$$
\begin{aligned}
(w_i - s^1 + \varepsilon) + m\left[(\varphi - 1)s^1 + s^1 - \varepsilon\right] &= (w_i - s^1) + m\varphi s^1 + (1 - m)\varepsilon \\
&> w_i - s^1 + m\varphi s^1
\end{aligned}
$$

since $m < 1$. Thus i has an incentive to deviate. It follows that $\tilde{c}_1 > 0$ as claimed.

To see that $c_1 > \varphi$, note that if $c_1 < \varphi$, player $i \in C_1$ is in the first group where the total contribution except for player i is S^1_{-i}. If she reduces her contribution from s^1 to $s^1 - \varepsilon > s^2$, she remains in the first group, but her payoff increases from $w_i - s^1 + m(S^1_{-i} + s_i)$ to

$$
w_i - s^1 + m(S^1_{-i} + s_i) + (1 - m)\varepsilon.
$$

Thus i has an incentive to deviate. This proves that $c_1 > \varphi$.

To verify that each C_1-player contributes fully, note that we now have $c_1 = D_1 \cdot \varphi + \widetilde{c}_1$, where $D_1 \geqslant 1$ and $\widetilde{c}_1 > 0$; hence, every C_1-player has a strictly positive probability of entering Group (D_1+1); that is, $\mathbb{P}_{s^1}(D_1+1) = \widetilde{c}_1/c_1 > 0$. Given a contribution profile $\boldsymbol{s}$ satisfying $s_i = s^1 < w_i$ for some $i \in C_1$, let $S = \varphi s^1$ be the total contribution in Group $1, \ldots, D_1$, and let $S' \leqslant \widetilde{c}_1 s^1 + (\varphi - \widetilde{c}_1)s^2$ be the total contribution in Group (D_1+1).[6] Then $S > S'$ since $s^1 > s^2$. Hence, if a C_1-player contributes $s_i = s^1 < w_i$, her payoff is

$$\begin{aligned}(w_i - s_i) + m&\left[\sum_{k=1}^{D_1} \mathbb{P}_{s^1}(k)S + \mathbb{P}_{s^1}(D_1+1)S'\right] \\ &= (w_i - s^1) + m\left[\big(1 - \mathbb{P}_{s^1}(D_1+1)\big)\, S + \mathbb{P}_{s^1}(D_1+1)S'\right] \\ &< (w_i - s^1) + mS.\end{aligned}$$

However, if she increases her contribution from s^1 to $s^1 + \varepsilon < w_i$, she enters the first group with certainty and obtains:

$$(w_i - s^1 - \varepsilon) + m\,(S + \varepsilon) = (w_i - s^1) + mS - (1-m)\varepsilon.$$

This deviation is profitable as long as ε is small enough. We thus proved that $s^1 = w_i$ if player i is in the first class.

(2). We first show that there is at least one High in C_1. Suppose this is not true; that is, suppose that $C_1 \subseteq N_L$. Then $s^1 = 1$ since each C_1-player contributes fully. We can show that in such a situation any C_2-player has an incentive to deviate. There are three cases to consider:

Case i: $\widetilde{c}_1 + c_2 \leqslant \varphi$. See Figure A.1(i). Since we assume that $C_1 \subseteq N_L$, there are more than $n_H > \varphi$ players outside of C_1, so that $|\mathcal{C}| \geqslant 3$ and $s^2 > 0$. In such a case, each C_2-player can reduce her contribution from s^2 to $s^2 - \varepsilon > s^3$ and remain in Group (D_1+1). By the same reasoning as in Lemma A.1(a), this is a profitable deviation.

Case ii: $\widetilde{c}_1 + c_2 > \varphi$ and $\widetilde{c}_1 + \widetilde{c}_2 = \varphi$. See Figure A.1(ii). Consider any player $i \in C_2$. If $s_i = s^2 < 1$, her payoff is

$$\begin{aligned}(w_i - s^2) + m&\left\{\mathbb{P}_{s^2}(D_1+1)\left[\widetilde{c}_1 + (\varphi - \widetilde{c}_1)s^2\right] + \big(1 - \mathbb{P}_{s^2}(D_1+1)\big)\,\varphi s^2\right\} \\ &< (w_i - s^2) + m\left[\widetilde{c}_1 + (\varphi - \widetilde{c}_1)s^2\right]\end{aligned}$$

because $\widetilde{c}_1 + (\varphi - \widetilde{c}_1)s^2 > \varphi s^2$. However, if she contributes $s^2 + \varepsilon < s^1$, she enters Group (D_1+1) with certainty and obtains

$$\begin{aligned}(w_i - s^2 - \varepsilon) &+ m\left[\widetilde{c}_1 + (\varphi - \widetilde{c}_1)s^2 + \varepsilon\right] \\ &= (w_i - s^2) + m\left[\widetilde{c}_1 + (\varphi - \widetilde{c}_1)s^2\right] - (1-m)\varepsilon,\end{aligned}$$

[6] We use a weak inequality here because it is not clear at this stage if there are players from classes after C_2 in group (D_1+1).

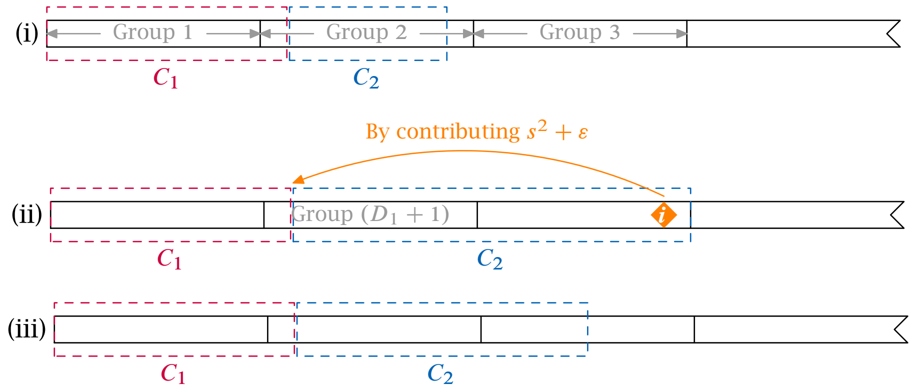


FIGURE A.1. There is at least one High in C_1

which is greater than her original payoff when ε is small enough. Thus, player $i \in C_2$ has an incentive to increase her contribution.

Case iii: $\tilde{c}_1 + c_2 > \varphi$ and $\tilde{c}_1 + \tilde{c}_2 \neq \varphi$. See Figure A.1(iii). This cannot be an equilibrium since any player $i \in C_2$ will increase her contribution for the same reason as in the former case.

Hence, there is at least one High i in C_1. Together with Lemma A.1(a) this implies that $s_i = H$. We thus conclude that $s^1 = H$ and $C_1 \subseteq N_H$.

(3). Suppose there exists a Class C_r satisfying $1 < s^r < H$. Since $s^r < H = s^1$, Class C_1 is ranked above Class C_r; since $s^r > 1$, there is at least one class after C_r and $C_r \subseteq N_H$. A similar argument as in Lemma A.1(b) shows that **(1)** C_1 is the immediate predecessor class of C_r, and **(2)** any C_r-player has an incentive to deviate. This proves the nonexistence of a Class C_r where $1 < s^r < H$.

(4). Let $\mathscr{C} = \{C_1, C_2\}$. Then $s^2 \leqslant 1$ because of the existence of Lows, and $N_L \subseteq C_2$ since $C_1 \subseteq N_H$ by Lemma A.1(a). Hence, $c_2 \geqslant n_L > \varphi$, $\tilde{c}_1 + c_2 > \varphi$ and $\tilde{c}_1 + \tilde{c}_2 = \varphi$, which is exactly Case ii) in Lemma A.1 (b); therefore, $s^2 = 1$ and $N_H \subseteq C_1$. This conclusion together with the fact that $C_1 \subseteq N_H$ implies that $C_1 = N_H$, and consequently $C_2 = N_L$.

(5). Let $c_R < \varphi$ and $s^R > 0$. Then Class C_R is in Group G, and each C_R-player gets $(w_i - s^R) + mS^G$, where S^G is the total contribution in Group G. If $i \in C_R$ reduces her contribution from s^R to 0, her payoff becomes $w_i + m(S^G - s^R) > (w_i - s^R) + mS^G$. Therefore, $s^R = 0$ in equilibrium when $c_R < \varphi$.

Let $\tilde{c}_R = 0$ and $s^R > 0$. Consider any C_R-player. If she reduces her contribution from s^R to 0, she enters the Group G, but is still grouped with $(\varphi - 1)$ players contributing s^R, so that her payoff increases by deviating this way. □

We are now ready to prove Theorem 4.

Proof of Theorem 4. It follows from Lemma 3 that $|\mathcal{C}| \geqslant 2$ in any equilibrium with positive contributions. Since $|\mathcal{C}| \leqslant n$ in any equilibrium, we can characterise the last class C_R, which can only take one of the following three forms:

1. $c_R = D_R \cdot \varphi + \tilde{c}_R$, where $D_R \geqslant 1$, and $\tilde{c}_R > 0$;
2. $c_R < \varphi$; or
3. $c_R = D_R \cdot \varphi$, where $D_R \geqslant 1$.

Also note that $s^R \leqslant 1$ in any equilibrium because of the existence of Lows. The proof will be given by the following four claims:

Claim 1. *If (1) holds, then the equilibrium candidate is a FEE.*

Let $c_R = D_R \cdot \varphi + \tilde{c}_R > \varphi$ with $\tilde{c}_R > 0$, and suppose that $s^R < 1$. Since $\tilde{c}_R > 0$ and $n = G\varphi$, we have $c_R \neq n$. So there exists at least one class C_{R-1} before C_R satisfying $s^{R-1} > s^R$. In this case, each C_R-player has an incentive to increase her contribution so that she can be grouped with the C_{R-1}-players with certainty. In equilibrium it must be that each C_R-player cannot increase her contribution further, i.e., $s^R = 1$ and $C_R \cap N_H = \varnothing$. Therefore, C_1 is the immediate predecessor class of C_R by Lemma A.1 (c), i.e., $|\mathcal{C}| = 2$. Lemma A.1 (d) implies that this is a FEE.

Claim 2. *If (2) holds, then the equilibrium candidate is a NEE.*

Suppose that $c_R < \varphi$ in equilibrium. In this case $|\mathcal{C}| \geqslant 3$ since $|\mathcal{C}| = 2$ implies that $c_R = n_L > \varphi$ by Lemma A.1 (d). Also note that $s^R = 0$ by Lemma A.1 (e). Consider Class C_{R-1}. There are three cases to consider:

Case i: $\boldsymbol{c_{R-1} + c_R \leqslant \varphi}$. See Figure A.2(i). This is impossible since C_{R-1} is in the last group and any C_{R-1}-player has an incentive to reduce her contribution for the same reason as in Lemma A.1 (e).

Case ii: $\boldsymbol{c_{R-1} + c_R > \varphi}$ and $\boldsymbol{\tilde{c}_{R-1} + \tilde{c}_R = \varphi}$. See Figure A.2(ii). With the following two steps we show that in this case $s_{R-1} = 1$:

Assume first that $s^{R-1} > 1$. Then Lows cannot be in C_{R-1} or the classes, if any, before C_{R-1} since $s^1 > \cdots > s^{R-1} > 1$, which means that $n_L \leqslant c_R < \varphi$. This contradicts Assumption B that $n_L > \varphi$.

Next assume that $s^{R-1} < 1$ and consider any player $i \in C_{R-1}$. If player i contributes $s_i = s^{R-1} < 1$, her expected payoff is

$$\begin{aligned} w_i - s^{R-1} + m \left\{ \left(1 - \mathbb{P}_{s^{R-1}}(G)\right) \varphi s^{R-1} + \mathbb{P}_{s^{R-1}}(G) \tilde{c}_{R-1} s^{R-1} \right\} \\ < w_i - s^{R-1} + m\varphi s^{R-1} \end{aligned}$$

since $\tilde{c}_{R-1} < \varphi$. But if she increases her contribution from s^{R-1} to $s^{R-1} + \varepsilon < \min\{1, s^{R-2}\}$, she enters the *first* group in class C_{R-1}, and gets

$$(w_i - s^{R-1} - \varepsilon) + m(\varphi s^{R-1} + \varepsilon) = (w_i - s^{R-1}) + m\varphi s^{R-1} - (1-m)\varepsilon,$$

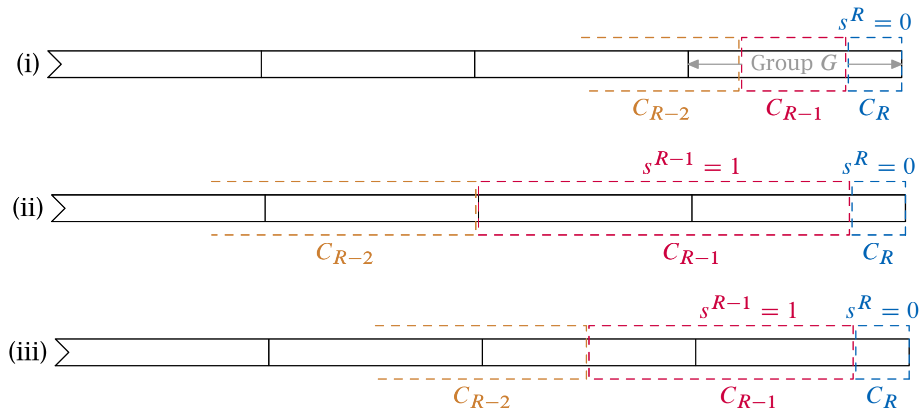


FIGURE A.2. The last class C_R

which is greater than her original payoff as long as ε is small enough.

The above two steps proved that $s^{R-1} = 1$ when $c_{R-1} + c_R > \varphi$ and $\widetilde{c}_{R-1} + \widetilde{c}_R = \varphi$. It follows from Lemma A.1 (c) that C_1 is the immediate predecessor class of C_{R-1}; that is, $|\mathcal{C}| = 3$. The fact that $\bigcup_{r=1}^{3} C_r = N$ implies:

$$
\begin{aligned}
n &= c_1 + c_2 + c_3 \\
&= (D_1 + D_2 + D_3)\,\varphi + \left(\widetilde{c}_1 + \widetilde{c}_2 + \widetilde{c}_3\right) \\
&\overset{\langle 1 \rangle}{=} (D_1 + D_2 + D_3)\,\varphi + \left(\widetilde{c}_1 + \varphi\right) \\
&= (D_1 + D_2 + D_3 + 1)\,\varphi + \widetilde{c}_1,
\end{aligned}
$$

where $\langle 1 \rangle$ holds because $\widetilde{c}_2 + \widetilde{c}_3 = \varphi$. The above equation implies that n is not a multiple of the group size φ because $0 < \widetilde{c}_1 < \varphi$ from Lemma A.1(a). This contradicts the assumption at the beginning of Section 3.1 that $n = G\varphi$, where $G \in \mathbb{N}$.

Case iii: $c_{R-1} + c_R > \varphi$ and $\widetilde{c}_{R-1} + \widetilde{c}_R \neq \varphi$. See Figure A.2(iii). In this case, $s^{R-1} = 1$ and $C_{R-1} \subseteq N_L$, otherwise any C_{R-1}-player will increase her contribution so that she can be grouped with C_{R-2}-players and avoid entering the last group. Lemma A.1 (c) implies that C_1 is the immediate predecessor class of C_{R-1}, i.e., $|\mathcal{C}| = 3$. We know the composition of the first two classes in terms of their members' endowments but we do not know for sure the composition of the third class, that we cannot exclude the possibility that $N_H \cap C_3 \neq \varnothing$ or that $N_L \cap C_3 \neq \varnothing$, so that $C_1 \subseteq N_H$ and $C_3 \subseteq N_H \cup N_L$.

Claim 3. *If (3) holds, then there is an equilibrium candidate, called E', which is not an equilibrium.*

Suppose that $c_R = D_R \cdot \varphi$. We first verify that $|\mathcal{C}| \neq 2$: if $|\mathcal{C}| = 2$, then $c_1 = n - c_2 = (G - D_2)\,\varphi$, which implies that $\widetilde{c}_1 = 0$, and contradicts Lemma A.1(a).

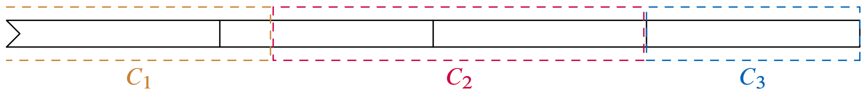


FIGURE A.3. $\tilde{c}_R = 0$

We next show that $|\mathcal{C}| = 3$ if $c_R = D_R \cdot \varphi$. Note that $|\mathcal{C}| \geqslant 3$ and $s^R = 0$ [Lemma A.1 (e)] imply $\tilde{c}_{R-1} > 0$ and $c_{R-1} > \varphi$, else any C_{R-1}-player has an incentive to reduce her contribution, which further implies that $s^{R-1} = 1$ and $C_{R-1} \subseteq N_L$ since each C_{R-1}-player wants to be grouped with C_{R-3}-players. Once again, Lemma A.1 (c) implies that C_1 is the immediate predecessor class of C_{R-1}; thus, the equilibrium structure is as in Figure A.3.

We will prove in Claim 4 that E' is not an equilibrium, but for now, we content ourselves with proving that $C_3 \subseteq N_L$: Suppose there exists a player i such that $i \in C_3 \cap N_H$. It follows that her payoff is H. But if she deviates and contributes $1+\varepsilon$, she enters group $(D_1 + 1)$, and since there exists at least one player contributing H in Group $(D_1 + 1)$ by Lemma A.1(a), player i can guarantee

$$(H - 1 - \varepsilon) + m[H + (\varphi - 2) + (1 + \varepsilon)] > H + (m\varphi - 1) - (1 - m)\varepsilon > H,$$

when $\varepsilon < (m\varphi - 1)/(1-m)$, where the first strict inequality holds because $H > 1$, and the second one *can* hold because $m\varphi > 1$. This proves that $C_3 \subseteq N_L$. Because $\bigcup_{r=1}^{3} C_r = N_H \cup N_L = N$, $C_2 \subset N_L$, and $C_3 \subset N_L$, we thus have $C_1 = N_H$ and $C_2 \cup C_3 = N_L$.[7]

Claim 4. *E' is not an equilibrium.*

E' is an equilibrium if and only if:

- Player $i \in C_3 \subseteq N_L$ has no incentive to increase her contribution from 0 to 1;
- Player $i \in C_2 \subseteq N_L$ has no incentive to reduce her contribution from 1 to 0;
- Player $i \in C_1 = N_H$ has no incentive to reduce her contribution from H to $1+\varepsilon$, 1, or 0, where $\varepsilon \to 0$.

Here below we examine the incentives of all players starting with the last class, and will show that there exists *no* equilibrium satisfying all these constraints.

Recall from Claim 3 that if E' is an equilibrium, we must have (i) $c_3 = D_3 \cdot \varphi$, and (ii) $c_2 + c_3 = n_L$ since $C_2 \cup C_3 = N_L$. Let $b = (B - D_3)\,\varphi$. This allows us to write c_2 as follows:

$$c_2 = n_L - c_3 = (B\varphi + \ell) - D_3 \cdot \varphi = b + \ell.$$

[7] More precisely, $i \in N_H \Rightarrow i \notin N_L \Rightarrow i \notin C_2 \cup C_3 \Rightarrow i \in C_1$, so that $N_H \subseteq C_1$. Combining this conclusion with the fact that $C_1 \subseteq N_H$ in Lemma A.1 (b) results in $C_1 = N_H$.

Incentives to Deviate for C_3-Players in E'. Consider any player $i \in C_3 \subseteq N_L$. Her payoff from contributing 0 is $U_0^L(C_3) = 1$. If i wants to deviate, she should contribute $s_i = 1$; then there would be $(c_2 + 1)$ players contributing 1, and i would enter Group $A+1, \dots, A+D_2+2$ with positive probabilities, which are:

$$\mathbb{P}_1(k) = \begin{cases} \ell/(c_2+1) & \text{if } k = A+1, \\ \varphi/(c_2+1) & \text{if } k = A+2, \dots, A+D_2+1, \\ 1/(c_2+1) & \text{if } k = A+D_2+2. \end{cases}$$

Because $\sum_{k=A+1}^{A+D_3+2} \mathbb{P}_1(k) = 1$, we have

$$\sum_{k=A+2}^{A+D_3+1} \mathbb{P}_1(k) = 1 - \mathbb{P}_1(A+1) - \mathbb{P}_1(A+D_3+2) = \frac{c_2 - \ell}{c_2+1} = \frac{b}{b+\ell+1}.$$

Recall that $S^{A+1} = hH + \ell$, so that player i's expected payoff from contributing 1 is

$$\begin{aligned} U_1^L(C_3) &= m\left[\mathbb{P}_1(A+1)S^{A+1} + \sum_{k=A+2}^{A+D_3+1} \mathbb{P}_1(k)\varphi + \mathbb{P}_1(A+D_3+2)\right] \\ &= m\left(\frac{\ell}{c_2+1}S^{A+1} + \frac{c_2-\ell}{c_2+1}\varphi + \frac{1}{c_2+1}\right) \\ &= \left(\frac{m}{b+\ell+1}\right)(\ell S^{A+1} + b\varphi + 1). \end{aligned}$$

Therefore, player $i \in C_3$ has no incentive to deviate if and only if $U_0^L(C_3) \geqslant U_1^L(C_3)$; that is

$$b \leqslant \frac{\ell + 1 - m\ell S^{A+1} - m}{m\varphi - 1}. \tag{A.1}$$

The above equation shows that there cannot be too many players in Class C_2 (recall that $c_2 = b + \ell$), else some players in class C_3 will have an incentive to try to go to C_2.

Incentives to Deviate for C_2-Players in E'. Consider any player $i \in C_2 \subseteq N_L$. If i contributes 1, she enters Group $A+1, \dots, A+D_2+1$ with positive probabilities, which are:

$$\mathbb{P}_1(k) = \begin{cases} \ell/c_2 & \text{if } k = A+1 \\ \varphi/c_2 & \text{if } k = A+2, \dots, A+D_2+1. \end{cases}$$

Her expected payoff is

$$U_1^L(C_2) = m\left(\frac{\ell}{c_2}S^{A+1} + \frac{c_2-\ell}{c_2}\varphi\right) = \left(\frac{m}{b+\ell}\right)(\ell S^{A+1} + b\varphi).$$

If $i \in C_2$ wants to deviate, she will contribute $\varepsilon \to 0$ in order to stay in Group $(A+D_2+1)$, and her expected payoff is

$$\lim_{\varepsilon\to 0} U_\varepsilon^L(C_2) = \lim_{\varepsilon \to 0}\left[1 + m(\varphi - 1) - (1-m)\varepsilon\right] = 1 + m(\varphi - 1).$$

Therefore, $i \in C_2$ has no incentive to deviate if and only if

$$U_1^L(C_2) \geqslant \lim_{\varepsilon \to 0} U_\varepsilon^L(C_2),$$

that is,

$$b \leqslant \frac{m\ell S^{A+1} - \left(1 + m\varphi - m\right)\ell}{1-m}. \tag{A.2}$$

The reason why b cannot be very large is as follows: Consider $i \in C_2$. If b is large, her probability of entering Group $(A+1)$ is small, and her expected payoff from contributing 1 is small, so that her incentive to deviate is large.

Note that by Claim 3, we also require $b \geqslant \varphi$, otherwise $i \in C_2$ will reduce her contribution. Combining this requirement, (A.1), and (A.2), we observe that m has to satisfy the following conditions:

$$\frac{\varphi + \ell}{\ell S^{A+1} - \ell\varphi + \varphi + \ell} \leqslant m \leqslant \frac{\varphi + \ell + 1}{\ell S^{A+1} + \varphi^2 + 1}. \tag{A.3}$$

The intuition behind (A.3) is as follows: m is the return from the group investment, so it cannot be very small because if it is very small C_2-players will have no incentive to contribute. At the same time, m cannot be very large because this would give C_3-players an incentive to contribute. These two constraints determine the bounds of m in (A.3).

For ease of expression, define

$$\frac{\varphi + \ell}{\ell S^{A+1} - \ell\varphi + \varphi + \ell} = \underline{m}, \quad \text{and} \quad \frac{\varphi + \ell + 1}{\ell S^{A+1} + \varphi^2 + 1} = \overline{m}.$$

It follows from (A.3) that $\underline{m} \leqslant \overline{m}$; thus given all other parameters, S^{A+1} must satisfy

$$S^{A+1} \geqslant \frac{-\ell^2 - \ell\varphi + \ell^2\varphi - \varphi^2 + 2\ell\varphi^2 + \varphi^3}{\ell}.$$

Substituting the above inequality to $\overline{m}$, we obtain

$$\overline{m} \leqslant \frac{1 + \ell + \varphi}{1 - \ell^2 - \ell\varphi + \ell^2\varphi + 2\ell\varphi^2 + \varphi^3}. \tag{A.4}$$

Incentives to Deviate for C_1-Players in E'. Note that $C_1 = N_H$ in E'. We have shown in Section 3.1 that a C_1-player has no incentive to reduce her contribution from H to $1 + \varepsilon$ if and only if

$$h \leqslant n_H \left(1 - \frac{M}{\ell}\right). \tag{6}$$

It can be seen that if (6) holds, then $1 - M/\ell > 0$, which means that

$$m > \frac{1}{\ell + 1}. \tag{A.5}$$

E' is not an equilibrium because (A.4) and (A.5) are incompatible: If E' is an equilibrium, m must satisfy $1/(\ell + 1) < m \leqslant \overline{m}$, so we must have $1/(\ell + 1) < \overline{m}$;

however,

$$\overline{m} - \frac{1}{\ell+1} \leqslant \frac{-\ell\left(\varphi-2\right)-\left(\varphi^2-\varphi\right)}{(1+\ell)\left[1+\ell(\varphi-1)+\varphi^2-\varphi\right]} < 0.$$

A contradiction.

Conclusion: Equilibrium candidate E' is not an equilibrium. □

Appendix B. The Results when Assumption B Fails

In this appendix, we discuss briefly the consequences when Assumption B fails.

Lemma B.1. *If there exists an equilibrium when* $0 < n_H < \varphi$*, then* $|\mathcal{C}| = 2$*,* $C_1 \subseteq N_L$*,* $C_2 \subseteq N_H \cup N_L$*,* $c_2 < \varphi$*,* $s^1 = 1$*, and* $s^2 = 0$*.*

Proof. By Lemma A.1 (a) and (b), $\tilde{c}_1 > 0$ and $c_1 > \varphi$ in equilibrium, so $s^1 = 1$ and $C_1 \cap N_H = \varnothing$. A similar argument as in Lemma A.1 (c) shows that there is no class C_r satisfying $0 < s^r < s^1 = 1$, so we conclude immediately that $|\mathcal{C}| = 2$ and $s^2 = 0$. Since $|\mathcal{C}| = 2$, we conclude that $N_H \subseteq C_2$; since $s^2 = 0$, we conclude that $c_2 < \varphi$, otherwise some player $i \in C_2$ will increase her contribution. □

Lemma B.2. *There is no equilibrium with positive contributions when* $n_H = \varphi$*.*

Proof. A similar argument as in Lemma B.1 shows that $|\mathcal{C}| = 2$, $s^1 = 1$, $s^2 = 0$, and $N_H \subseteq C_1$. Since $\tilde{c}_1 > 0$, there are some low types contribute nothing, which means that there are more than φ players in class C_2. Therefore, there does not exist an equilibrium with positive contributions in this case. □

The cases that $0 < n_L < \varphi$ and $n_L = \varphi$ are easy to analyze, and I omit the proofs.

Lemma B.3. *If there is an equilibrium when* $n_H = A\varphi$ *and* $A \in \{2, 3, \ldots, G-2\}$*, then* $|\mathcal{C}| = 3$*,* $C_1 \subsetneq N_H$*,* $C_2 \subseteq N_L$*,* $C_3 \subseteq N_L \cup N_H$*,* $\tilde{c}_1 > 0$*,* $c_2 + c_3 > \varphi$*,* $\tilde{c}_2 + \tilde{c}_3 > 0$*,* $c_3 < \varphi$*,* $s^1 = H$*,* $s^2 = 1$*, and* $s^3 = 0$*.*

Proof. This case is not very special. By Lemma A.1 (a), $\tilde{c}_1 > 0$ and $c_1 > \varphi$, so some players from N_H are not in C_1 and $s_2 = 1$. Because $s_2 = 1$, we know $C_2 \cup N_H = \varnothing$ and $s_i = 0$ for all $i \in N_H \setminus C_1$, which implies $i \in C_3$ for all $i \in N_H \setminus C_1$ and $s_3 = 0$. We have shown in Claim 4 that $\tilde{c}_3 = 0$ is impossible, so $c_3 < \varphi$. □

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