1. INTRODUCTION

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1. Pure Strategies

1.1. The Battle of the Sexes (BOS, for abbreviation)

1.1.1. The Payoff Matrix

We can use Figure 1.1 to represent the BOS game. The rows of the table represent the possible actions B ["boxing"] and O ["Opera"] for *Husband*, and the columns represent the actions B and O of *Wife*.

In each cell, the upper left entry (the blue one) is the *payoff* to *Husband*, while the lower right entry (the red one) is the payoff to *Wife*.

Husband \ Wife	B	O
B	3, 1	0, 0
O	0, 0	1, 3

Figure 1.1: The Battle of the Sexes

1.1.2. How to Find A Nash Equilibrium:

Definition 1.1 (Nash equilibrium)***.*** A pair of players' actions that are best responses to each other.

To find a Nash equilibrium of BOS, we need to identify each player's best response to each of the other's actions. We could start by identifying *Husband*'s best responses to each of *Wife*'s two possible actions. Then we could identify *Wife*'s best responses to each of *Husband*'s two possible actions. If any two of the actions we identify are best responses to each other then we will have found an action pair that constitutes a Nash equilibrium.

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The trick then is to identify both players' best response actions. The way we will do this here is by drawing boxes on the payoffs corresponding to each player's best response to each of the actions of the other. If we follow this procedure for each player then any cell where both payoffs are boxed will identify a Nash equilibrium.

Step 1 If *Wife* chooses B, then *Husband*'s best response is to choose B; by choosing B his payoff is 3 whereas he only gets 0 by choosing O. So in the Matrix BOS-1 I have boxed *Husband*'s payoff 3.

Wife

Husband		***B***	
	B	[3]	
	O	0	

BOS-1

Step 2 If *Wife* chooses O then *Husband*'s best response is O. So I have boxed his payoff 1 in the Matrix BOS-2.

Wife

Husband			***O***
	B		0
	O		[1]

BOS-2

Step 3 If *Husband* chooses B then *Wife*'s best response is B. So I have boxed her payoff 1 in BOS-3.

Step 4 If *Husband* chooses O then *Wife*'s best response is O. So I have boxed her payoff 3 in BOS-4.

Step 5 In Matrix BOS-5, both players' best response payoffs are boxed. Find the cells with two boxes. The Nash equilibria in this game are

$$(B, B) \quad \text{and} \quad (O, O).$$

Wife

Husband	***B***	*O*
B	1	0

BOS-3

Wife

Husband	*B*	***O***
O	0	3

BOS-4

1.2. **Matching Pennies: There is no (pure) NE because there are no action pairs where the actions are best responses to each other**

1.3. **Find the (pure) Nash equilibria in the game of chicken and stag hunt**

Wife

Husband	*B*	*O*
B	3, 1	0, 0
O	0, 0	1, 3

BOS-5

Player 2

Player 1	H	T
H	$\boxed{1}$, -1	-1, $\boxed{1}$
T	-1, $\boxed{1}$	$\boxed{1}$, -1

Matching Pennies

Player 2

Player 1	*Swerve*	*Straight*
Swerve	0, 0	-1, 1
Straight	1, -1	-2, -2

The Game of Chicken

1.4. Rock, Paper, Scissors: There is no NE

2. Mixed Strategies

Example 2.1. If a player is choosing between two actions called *left* and *right*, one possible mixed strategy would be to choose *left* with probability 1/4 and *right* with probability 3/4. This particular mixed strategy could then be written in shorthand either as

$$\left(\frac{1}{4}, \frac{3}{4}\right)$$

or

$$\frac{1}{4}\textit{left} + \frac{3}{4}\textit{right}.$$

Hunter 2

Hunter 1	*Stag*	*Rabbit*
Stag	2, 2	0, 1
Rabbit	1, 0	1, 1

Stag Hunt

Child 2

Child 1		*Rock*	*Paper*	*Scissors*
	Rock	0, 0	-1, $\boxed{1}$	$\boxed{1}$, -1
	Paper	$\boxed{1}$, -1	0, 0	-1, $\boxed{1}$
	Scissors	-1, $\boxed{1}$	$\boxed{1}$, -1	0, 0

Rock, Paper, Scissors

2.1. BOS

The intuitive argument goes like this: if either player chooses a mixed strategy then they must be *indifferent* between playing either of their pure actions. If not, then one pure action would be preferred and they would choose that rather than randomising [maybe an example is needed here]. According to this logic for a mixed strategy to be part of a Nash equilibrium for *Husband*, he must be indifferent between choosing B or O. If this is the case his expected payoff from choosing B must be the same as his expected payoff from choosing O. Similarly, if *Wife* chooses a mixed strategy, her expected payoff from choosing B must be the same as her expected payoff from B.

Wife

Husband		B $[x]$	O $[1-x]$
	B $[y]$	3, 1	0, 0
	O $[1-y]$	0, 0	1, 3

BOS

Step 1

- Given *Wife*'s mixed strategy $(x, 1-x)$, *Husband* gets

$$3 \cdot x + 0 \cdot (1-x) = 3 \cdot x \quad \text{if he plays } B$$
$$0 \cdot x + 1 \cdot (1-x) = 1 - x \quad \text{if he plays } O.$$

- *Husband* must be *indifferent* between B and O; hence

$$3 \cdot x = 1 - x,$$

which solves for $x = \frac{1}{4}$.

- Therefore, *Wife*'s mixed strategy is

  $(x, 1-x) = \left(\frac{1}{4}, \frac{3}{4}\right)$: *Wife* plays B with probability $1/4$, and plays O with probability $3/4$.

Step 2

- Given *Husband*'s mixed strategy $(y, 1-y)$, *Wife* gets

$$\begin{aligned} 1 \cdot y + 0 \cdot (1-y) &= 1 \cdot y = y && \text{if she plays } B \\ 0 \cdot y + 3 \cdot (1-y) &= 3 \cdot (1-y) && \text{if she plays } O. \end{aligned}$$

- *Wife* must be *indifferent* between B and O; hence

$$y = 3 \times (1-y)$$

  which solves for $y = \frac{3}{4}$.

- Therefore, *Husband*'s mixed strategy is

  $(y, 1-y) = \left(\frac{3}{4}, \frac{1}{4}\right)$: Husband plays B with probability $\frac{3}{4}$, and plays O with probability $\frac{1}{4}$.

Step 3 In sum, the mixed NE in this game is

$$\left(\underset{\text{Husband}}{\left(\frac{3}{4}, \frac{1}{4}\right)}, \underset{\text{Wife}}{\left(\frac{1}{4}, \frac{3}{4}\right)} \right).$$

Sometimes, we write the mixed NE as

$$\left(\frac{3}{4}B + \frac{1}{4}O, \frac{1}{4}B + \frac{3}{4}O\right).$$

2.2. Matching Pennies

		Child 2: H $[x]$	Child 2: T $[1-x]$
Child 1	H $[y]$	1, -1	-1, 1
	T $[1-y]$	-1, 1	1, -1

Matching Pennies

Step 1

- Given Player 2' mixed strategy $(x, 1-x)$, Player 1 gets

  $\underline{\hspace{5cm}}$ if he plays H

  $\underline{\hspace{5cm}}$ if he plays T

- Player 1 must be indifferent between H and T; hence

  $$\underline{\hspace{5cm}},$$

  which solves for $x = \underline{\hspace{2cm}}$.

- Therefore, Player 2's mixed strategy is $\underline{\hspace{2cm}}$: he plays H with prob. $\underline{\hspace{1cm}}$, and plays T with prob. $\underline{\hspace{1cm}}$.

Step 2

- Given Player 1' mixed strategy $(y, 1-y)$, Player 2 gets

  $\underline{\hspace{5cm}}$ if he plays H

  $\underline{\hspace{5cm}}$ if he plays T

- Player 2 must be indifferent between H and T; hence

  $$\underline{\hspace{5cm}},$$

  which solves for $y = \underline{\hspace{1cm}}$.

- Therefore, Player 1's mixed strategy is $\underline{\hspace{2cm}}$: he plays H with prob. $\underline{\hspace{1cm}}$, and plays T with prob. $\underline{\hspace{1cm}}$.

Step 3 In sum, the mixed NE in this game is

$$\underbrace{\underline{\hspace{2cm}}}_{\text{Player 1}}, \underbrace{\underline{\hspace{2cm}}}_{\text{Player 2}}$$

We can also express the mixed NE as $\underline{\hspace{5cm}}$

2.3. Rock, Paper, Scissors

Step 1

- Given Player 2's mixed strategy $(a, b, 1-a-b)$, Player 1 gets

$$\begin{cases} [0 \cdot a] + \big[(-1) \cdot b\big] + \big[1 \cdot (1-a-b)\big] = 1 - a - 2b & \text{if he plays } \textit{Rock} \\ [1 \cdot a] + [0 \cdot b] + \big[(-1) \cdot (1-a-b)\big] = 2a + b - 1 & \text{if he plays } \textit{Paper} \\ \big[(-1) \cdot a\big] + [1 \cdot b] + \big[0 \cdot (1-a-b)\big] = -a + b & \text{if he plays } \textit{Scissors.} \end{cases}$$

		Child 2		
		Rock $[a]$	Paper $[b]$	Scissors $[1-a-b]$
Child 1	Rock $[c]$	0, 0	-1, 1	1, -1
	Paper $[d]$	1, -1	0, 0	-1, 1
	Scissors $[1-c-d]$	-1, 1	1, -1	0, 0

Rock, Paper, Scissors

- Player 1 should be indifferent among *Rock*, *Paper*, and *Scissors*; therefore,

$$\begin{cases} 1-a-2b = 2a+b-1, & \text{Player 1 is indifferent between } \textit{Rock} \text{ and } \textit{Paper} \\ 1-a-2b = -a+b, & \text{Player 1 is indifferent between } \textit{Rock} \text{ and } \textit{Scissors} \end{cases}$$

Subtract the second equation from the first equation, we get

$$0 = 3a-1,$$

which solves for $a = \frac{1}{3}$. Input $a = \frac{1}{3}$ into the second equation,[1] and we get

$$b = \frac{1}{3};$$

hence,

$$1-a-b = 1-\frac{1}{3}-\frac{1}{3} = \frac{1}{3}.$$

- Therefore, Player 2's mixed strategy is

$$(a, b, 1-a-b) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

Step 2

- Given Player 1's mixed strategy $(c, d, 1-c-d)$, Player 1 gets

______________________________ if he plays *Rock*

______________________________ if he plays *Paper*

______________________________ if he plays *Scissors*

[1] Actually, we can get $b = \frac{1}{3}$ from the second equation immediately; however, here I provide a general way so that you can solve such kind of problems.

- Player 1 should be indifferent among *Rock*, *Paper*, and *Scissors*; therefore,

  ____

  Solve this problem, and we get

  $$(c, d, 1 - c - d) = \underline{\hspace{6cm}}.$$

Step 3 In sum, the mixed NE in this game is

$$\underline{\hspace{7cm}}.$$