10. Applications

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Exercise 1.

Step 1 In this game:

- The set of players is $N = \{L, R\}$;
- The sets of strategies are $S_1 = \{x \in \mathbb{R} : x \geqslant 0\}$ and $S_2 = \{y \in \mathbb{R} : y \geqslant 0\}$;
- The payoff function for player L is

$$u_L(x, y) = \begin{cases} \dfrac{x}{x+y} - x, & \text{if } x > 0 \\ 0, & \text{if } x = 0, \end{cases}$$

and the payoff function for player R is

$$u_R(x, y) = \begin{cases} \rule{5cm}{0.4pt} \\ \rule{5cm}{0.4pt}. \end{cases}$$

Step 2 For player L, if $y = 0$,

$$u_L(x, 0) = \begin{cases} 0, & \text{if } x = 0 \\ 1 - x, & \text{if } x > 0. \end{cases}$$

Hence, when $y = 0$, player L will choose a small positive x [of course, $x < 1$].

Step 3 Now let $y > 0$. Player L's problem becomes

$$\max_{x \geqslant 0} \left\{ \frac{x}{x+y} - x \right\}.$$

The first order condition gives player L's best response function

$$x = \sqrt{y} - y,$$

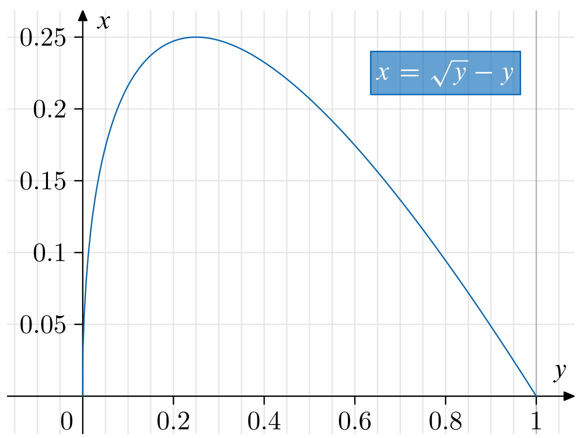

Figure 0.1: Player L's best response function when $y > 0$

which is depicted in Figure 0.1.

Hence, player L's best response function is

$$x = \begin{cases} \text{a small positive real number}, & \text{if } y = 0 \\ \sqrt{y} - y, & \text{if } y > 0. \end{cases}$$

Step 4 Similarly, player R's best response function is

$$y = \begin{cases} \rule{5cm}{0.4pt} \\ \rule{5cm}{0.4pt}. \end{cases}$$

In Figure 0.2 we put these two best response functions together.

Step 5 To find the NEs, we can just solve the following system of equations:

$$\begin{cases} x = \sqrt{y} - y \\ y = \sqrt{x} - x. \end{cases}$$

The solution is

$$x = y = \frac{1}{4}.$$

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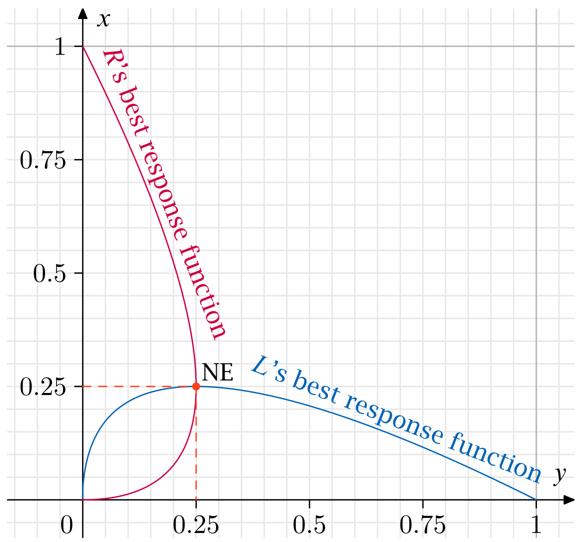

Figure 0.2: Best response functions

Exercise 2.

Step 1 X's problem is:

$$\max_{p_x} \left\{ \left(44 - 2p_x + p_y\right) \cdot \left(p_x - 8\right) \right\}.$$

Step 2 The first order condition gives X's best response function

$$p_x = 15 + \frac{p_y}{4}.$$

Step 3 Y's problem is:

Step 4 The first order condition gives Y's best response function

$$p_y = 14 + \frac{p_x}{4}.$$

Step 4 To find the NE, we need only to solve the following system of equations:

$$\begin{cases} p_x = 15 + \dfrac{p_y}{4} \\ p_y = 14 + \dfrac{p_x}{4}. \end{cases}$$

In Figure 0.3 we put these two best response functions together. The solution is

$$\begin{cases} p_x = \underline{\hspace{3cm}} \\ p_y = \underline{\hspace{3cm}}. \end{cases}$$

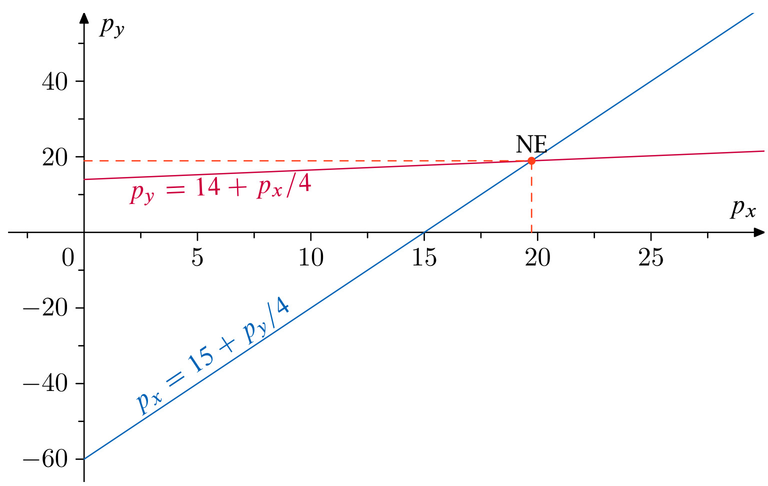

Figure 0.3: Best Response Function

> *Remark.* Now you should know how to solve Exercise 3. We will talk about it in class.