6. EXTENSIVE FORM GAMES WITH PERFECT INFORMATION

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Exercise 1. The player who brings the total to 100 wins; therefore if a players has to move and the current total is an integer in the interval $(90, 99)$ he can win by choosing a number equal to 100 minus the current total [e.g., if the current total is 93, then he can choose $100 - 93 = 7$ and win]. We continue by steps:

(a) If the current total is equal to 89 the player who has to move loses because he would bring the total to the interval $(90, 99)$. [This is because $90 \leqslant 89 + x \leqslant 99$ if $1 \leqslant x \leqslant 10$.]
(b) If the current total is in the interval $(79, 88)$ the player who has to move wins because he can bring the total to 89 and, consequently, his opponent will lose.
(c) If the current total is equal to 78 the player who has to move loses because he would bring the total to the interval $(79, 88)$.
(d) If the current total is in the interval $(68, 77)$ the player who has to move wins because he can bring the total to 78 and, consequently, his opponent will lose.
(e) $\cdots$
(f) If the current total is equal to 12 the player who has to move loses because he would bring the total to the interval $(13, 22)$ and, consequently, his opponent will lose.
(g) If the current total is in the interval $(2, 11)$ the player who has to move wins because he can bring the total to 12 and, consequently, his opponent will lose.
(h) If the current total is equal to 1 the player who has to move loses because he would bring the total to the interval $(2, 11)$ and, consequently, his opponent will lose.
(i) Player 1 chooses first. Therefore, in a backwards induction equilibrium player 1 wins by choosing the number 1 in the first stage.

Exercise 2.

> *Remark.* In Figure 0.1, each decision node has two labels, separated by a decimal point. To the left of the decimal point, we write the *player label*, which indicates the name of the layer who controls the node.

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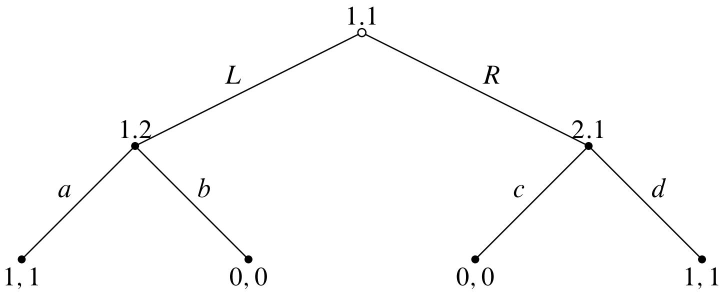

Figure 0.1: Game 2-1

(a).

Step 1 Player 1 chooses a at 1.2, and Player 2 chooses d at 2.1 [see Figure 0.2]. Hence, the Game 2-1 becomes Game 2-3 [see Figure 0.3].

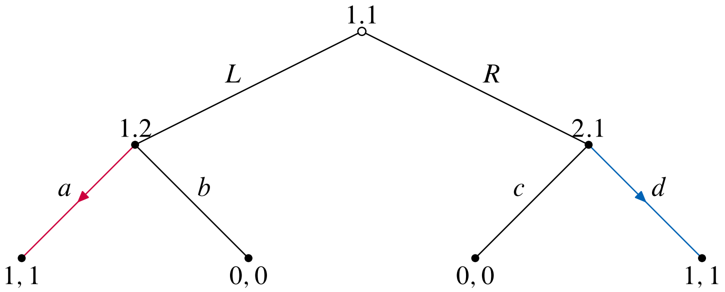

Figure 0.2: Game 2-2

Step 2 Player 1 is indifferent between L and R since he always gets 1, so any choice at point 1.1 is reasonable. Denote Player 1's choice at point 1.1 as

$$x \cdot L + (1 - x) \cdot R, \quad x \in [0, 1],$$

which means that Player 1 plays L with probability x, and plays R with probability $1 - x$.

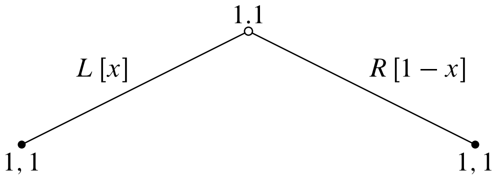

Figure 0.3: Game 2-3

Step 3 Therefore, the set of SPNE is [note that I add a red diamond between the two strategies, and an underline below Player 1's strategy so that you can read them easier]:

$$\left\{\underline{x\cdot L+(1-x)\cdot R, a}, \ \diamond\ d\right\}, \quad x\in[0,1].$$

(b). Player 1's set of strategies is

$$S_1=\{La, Lb, Ra, Rb\};$$

Player 2's set of strategies is

$$S_2=\{c, d\}.$$

		Player 2	
		c	d
Player 1	La	1 1	1 1
	Lb	0 0	0 0
	Ra	0 0	1 1
	Rb	0 0	1 1

(c). The set of pure Nash equilibria is

$$NE^p=\{(La,c),(La,d),(Ra,d),(Rb,d)\}.$$

However, Ra and Rb are Player 1's weakly dominated strategies [both dominated by La], and c is Player 2's weakly dominated strategy [dominated by d], so the set of admissible equilibria, AE, is

$$AE=\{(La,d)\}.$$

Exercise 3.

Step 1 At the subgame beginning at 1.2, Player 1's best response is g since $4>0$ [see Figure 0.5].

Step 2 At the subgame beginning at 2.1, Player 2's best response is d since $2>1$ [see Figure 0.6].

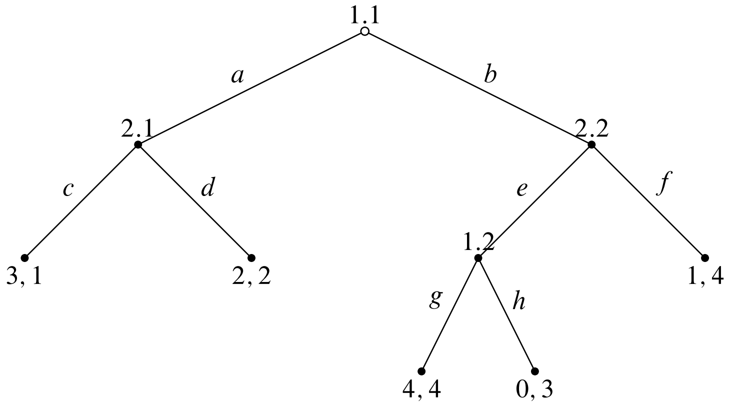

Figure 0.4: Game 3-1

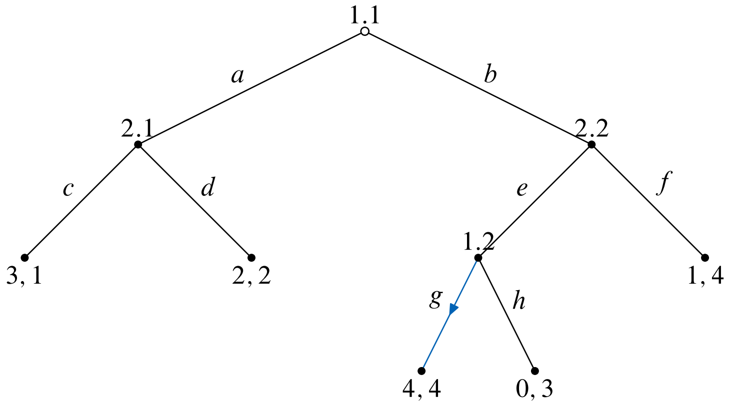

Figure 0.5: Game 3-2

Step 3 At the subgame beginning at 2.2, any of Player 2's choice is reasonable since he always gets 4. Let Player 2's choice be

$$x \cdot e + (1 - x) \cdot f, \quad x \in [0, 1] .$$

[See Figure 0.7.]

Step 4 Now the game becomes as in Figure 0.8, where Player 1's payoff is derived as follows

$$4 \cdot x + 1 \cdot (1 - x) = 3x + 1.$$

Then there are three cases:

Case 1 $2 = 3x + 1$, that is, $x = \frac{1}{3}$;

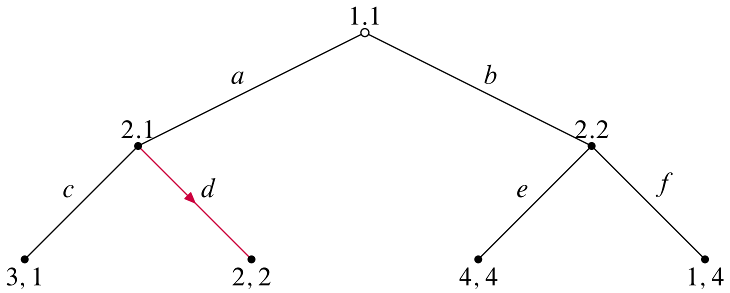

Figure 0.6: Game 3-3

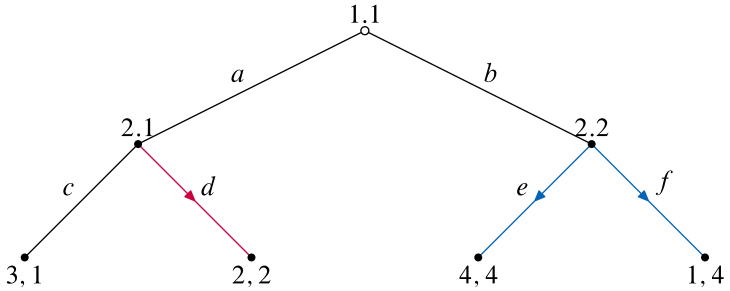

Figure 0.7: Game 3-4

Case 2 $2 > 3x + 1$, that is, $0 \leqslant x < \frac{1}{3}$;

Case 3 $2 < 3x + 1$, that is, $\frac{1}{3} < x \leqslant 1$.

Step 5 If $x = \frac{1}{3}$, Player 1 is indifferent between a and b at point 1.1; see Figure 0.9; if $0 \leqslant x < \frac{1}{3}$, then Player 1's best response is a at point 1.1; finally, if $\frac{1}{3} < x \leqslant 1$, then Player 2's best response is b at point 1.1.

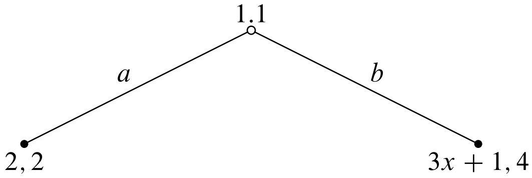

Figure 0.8: Game 3-5

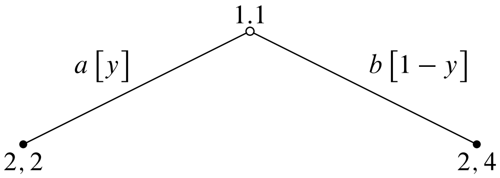

Figure 0.9: Game 3-6

Step 6 Hence, the set of SPNE is

$$\left\{\underline{y\cdot a+(1-y)\cdot b, g,}\ \diamond\ \underline{\frac{1}{3}e+\frac{2}{3}f, d}: 0\leqslant y\leqslant 1\right\}\cup$$
$$\left\{\underline{a, g,}\ \diamond\ \underline{x\cdot e+(1-x)\cdot f, d}: 0\leqslant x<\frac{1}{3}\right\}\cup$$
$$\left\{\underline{b, g,}\ \diamond\ \underline{x\cdot e+(1-x)\cdot f, d}: \frac{1}{3}<x\leqslant 1\right\}.$$