8. Forward induction

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- **Backword induction:** past behavior is inferred from future behavior.
- **Forward induction:** future behavior can be inferred from past behavior in some way—A player looks back up the tree, to take account of moves that another player could have made but didn't make, to try to predict what that player will do subsequently. That is, the player tries to "induce forwards" from other players' past behavior to their future behavior.

Exercise 1.

Extensive Form Representation

Step 1 Player N (*Nature*) moves first; he decides Player 1's type: s (*Surly*) with probability 0.9, and w (*Wimp*) with probability 0.1. See Figure 0.1.

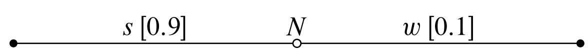

Figure 0.1: Nature's moves

Step 2 Now Player 1 moves. He has two actions at each of his decision nodes (information sets): B (*Beer*) and Q (*Quiche*). See Figure 0.2.

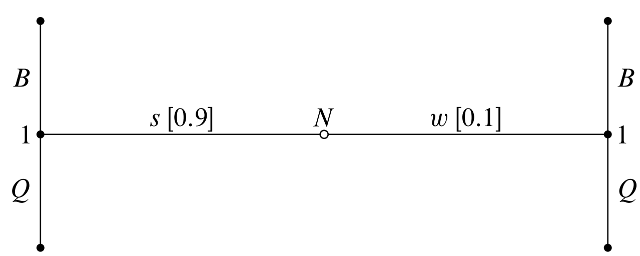

Figure 0.2: Player 1's moves

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Step 3 When Player 2 moves, he does not observe Player 1's types. Use the *information sets* to denote this fact in Figure 0.2.

Step 4 For Player 2, he has two actions at each of his information set: D (*Duel*) and ND (*Not Duel*). Now you can complete the extensive form game in Figure 0.3.

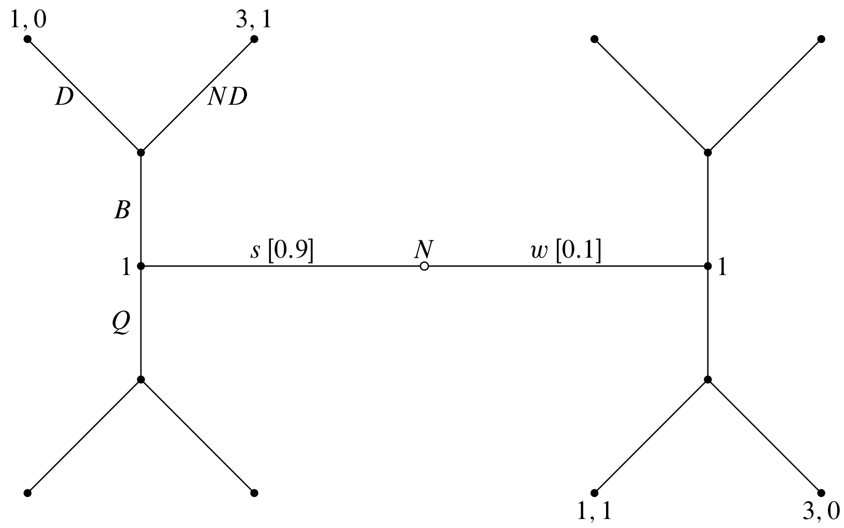

Figure 0.3: The extensive form game

Remark. Note that Figure 0.4 is also acceptable.

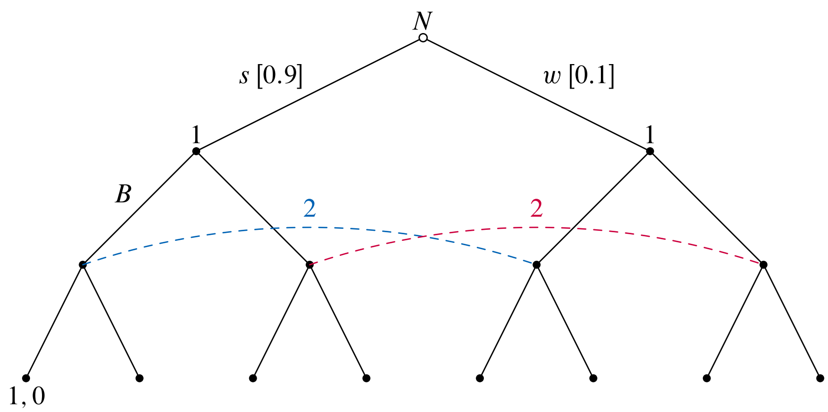

Figure 0.4: An alternative expression

Normal Form Representation

Step 1 The set of players is $\{1, 2\}$; Player 1's set of pure strategies is

$$S_1 = \{B - B, B - Q, Q - B, Q - Q\}$$

[where, for example, strategy $B - Q$ means that Player 1 has *beer* if his type is *surly*, and has *quick* if his type is *wimp*]; Player 2's set of pure strategies is [Be sure of what you are writing. Basically, we assume that in Player 2's strategies, the first element is his action if Player 1 has *beer*, and the second element is his action if Player 2 has *quick*.]:

$$S_2 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Step 2 We also need to know the pairs of payoffs. For instance, if the pair of strategies is $(B - B, D - D)$, then Player 1's payoff is

$$0.9 \times 1 + 0.1 \times 0 = 0.9,$$

and Player 2's payoff is

$$0.9 \times 0 + 0.1 \times 1 = 0.1.$$

Step 3 With step 1 and 2, now you can complete the following matrix.

Player 2

Player 1		$D - D$			
	$B - B$	0.9, 0.1			

Matrix 1

Nash Equilibrium

We know from Matrix 1 that there are two pure strategy Nash equilibria:

$$(B - B, ND - D) \quad \text{and} \quad \left(Q - Q, D - ND\right).$$

Subgame Perfect Nash Equilibrium

The point here is *whether there are any proper subgames in the whole game*? You can get some hints from Figure 0.4.

Forward Induction

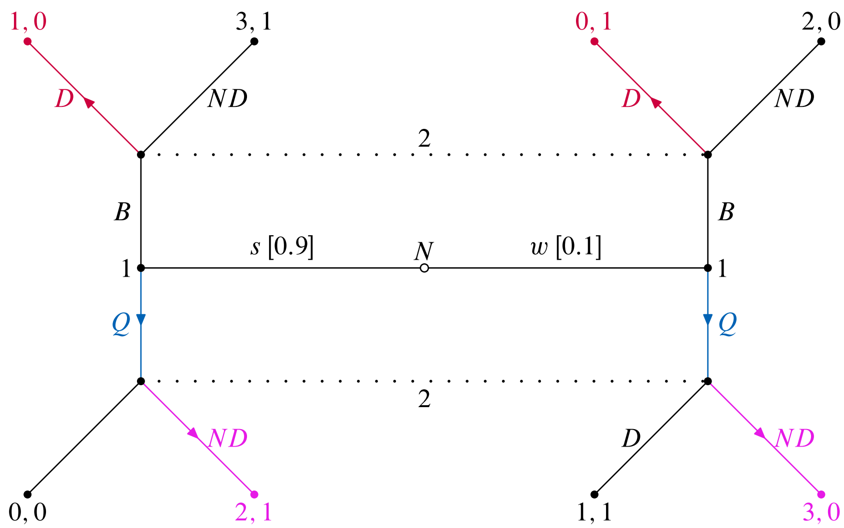

Figure 0.5: Equilibrium $(Q-Q, D-ND)$

Equilibrium $(Q-Q, D-ND)$.

Step 1 Does the w-type have any incentives to deviate from Q? Why?

Step 2 Does the s-type have any incentives to deviate from Q? The answer is "Yes" if Player 2 plays ND upon observing B since in this case, Player 1 gets 3, which is greater that 2, the payoff he gets in equilibrium. Note that Player 2 will play ND if he is sure that Player 1's type is s $(1 > 0)$.

Step 3 Therefore, the problem is if Player 1 can persuade Player 2 to believe that his type is s when his type is really s by having beer.

Equilibrium $(B-B, ND-D)$. This equilibrium satisfies forward induction. Why? [Hint: Does the s-type have incentive to deviate from B? How about the w-type?]

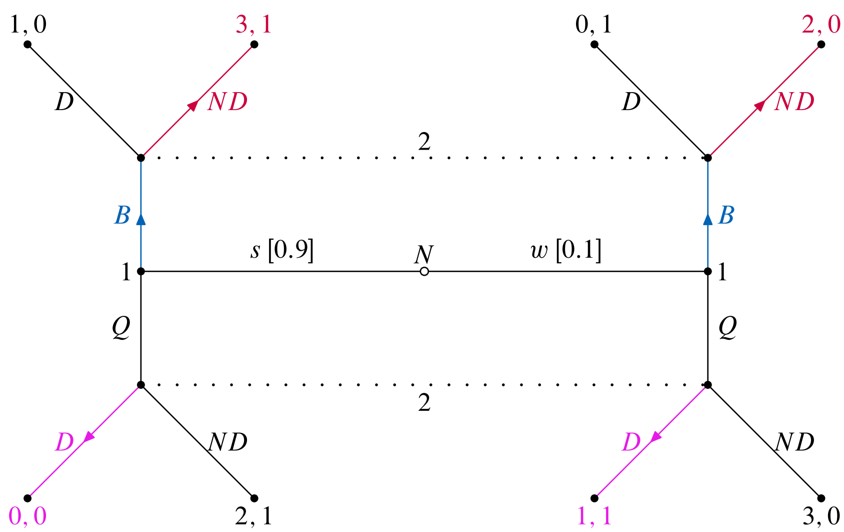

Figure 0.6: Equilibrium $(B - B, ND - D)$