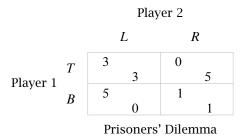
9. REPEATED GAMES

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Exercise 1 (Prisoners' Dilemma).



Step 1 In an isolated interaction, (B, R) is the strictly dominant strategy equilibrium.

Step 2 To find the minimum discount factor δ , we use the *grim trigger strategies* [notice that min max u_i (s_1, s_2) = 1, $\forall i = 1, 2$]:

- Cooperate in the first period and to continue to do so in every subsequent period as long as both players have previously cooperate,
- while playing *B* and *R* in all other circumstances.

Step 3 Given the grim trigger strategies, if Player 1 cooperates, his flow of payoffs is

Time123 \cdots Payoff333 \cdots

but if he deviates at time 1, his flow of payoffs is

[By the *one-shot deviation principle*, we need only to consider such a deviation.]

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Step 4 Hence, the condition is

$$+3\cdot\delta+3\cdot\delta^2+\cdots \ge$$

that is,

3

which solves for

$$\frac{3}{1-\delta} \ge 5 + \frac{\delta}{1-\delta},$$
$$\delta \ge \frac{1}{2}.$$

Theorem 0.1. If $\delta \in (0, 1)$, then

$$a + a \cdot \delta + a \cdot \delta^2 + a \cdot \delta^3 + \dots = \frac{a}{1 - \delta}.$$

 $X = a + a \cdot \delta + a \cdot \delta^2 + a \cdot \delta^3 + \cdots,$

Proof. Let

then

$$X \cdot \delta = a \cdot \delta + a \cdot \delta^2 + a \cdot \delta^3 + \cdots$$

Thus,

$$\begin{aligned} X - X \cdot \delta &= (1 - \delta) \cdot X \\ &= a \end{aligned}$$

as $a \cdot \delta^{\infty} \to 0$ since $\delta < 1$. The above equation solves for

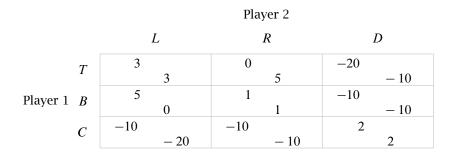
$$X = \frac{a}{1-\delta}.$$

Exercise 2 (Repeated Twice).

Step 1 There are two pure strategy Nash equilibria of the one-shot game:

$$(B, R)$$
 and (C, D) .

Step 2 Any SPNE involves playing either of these two pure strategy Nash equilibria in the second period.



Step 3 With Step 1 and 2, you now should know whether (T, L) is possible in the first stage. [*Hint*: Consider Player 2's strategy: play *L* in the first stage; play *D* in the second stage if Player 1 played *T* in the first stage, otherwise play *R*. Does Player 1 have incentive to cooperate at the first stage?]

Exercise 3 (Repeated Three Times). *Hint*: Use the Nash equilibrium at the second and third stage.